El problema inverso en la astrofísica de hoyos negros

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Outline

Description of an inverse problem

Various examples

Brutal force vs intelligence methods

Inverse / Direct problems involving BHs

- Binary black holes from the strain
- Kicked black hole
- Wandering black hole
- Intrinsic parameters out of an image

Direct and Inverse problem

Consider a phenomenon described by

$$given\ x\ define\ F(x) =: y$$

Here x may represent initial conditions or physical parameters.

And F represents the model or theory assumed to rule the phenomenon.

Direct problem. Given x and F calculate y. Easy $y = F(x)$.

Inverse problem of the cause. Given F and y, find x that produces y. It could be $x = F^{-1}(y)$

Inverse problem of the model. Given y and values x, find F such that $F(x)$ produces y.

Inverse problem of the cause and the model. Given y, find F and x such that $F(x)$ produces y.

Examples of direct problem

Given m, b, k, $x(0)$ and $\dot{x}(0)$ determine $x(t)$ when it obeys

$$m\ddot{x} + b\dot{x} + kx = 0$$

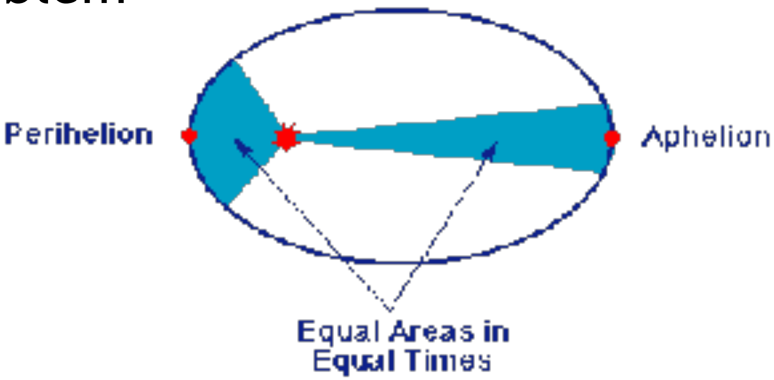
Given the initial position $\mathbf{x}(0)$ of a particle of mass m moving around an object of mass M, calculate $\mathbf{x}(t)$ for $t > 0$, provided

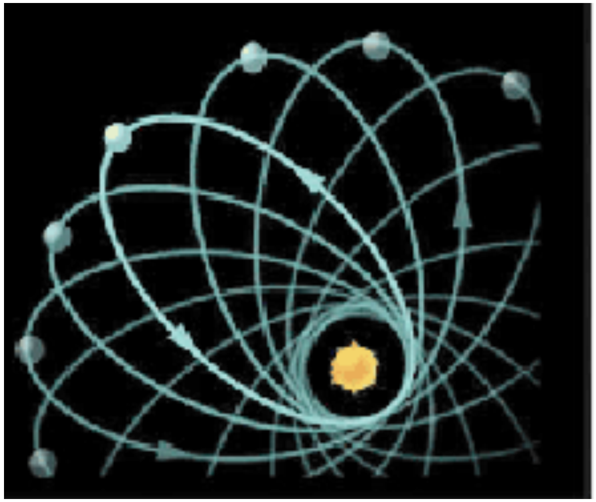
$$\mathbf{F} = G\frac{Mm}{|\mathbf{x}|^3}\mathbf{x} = m\ddot{\mathbf{x}}$$

These are simple PVIs

Examples of Inverse problems / data fitting

Kepler's problem / Newton problem

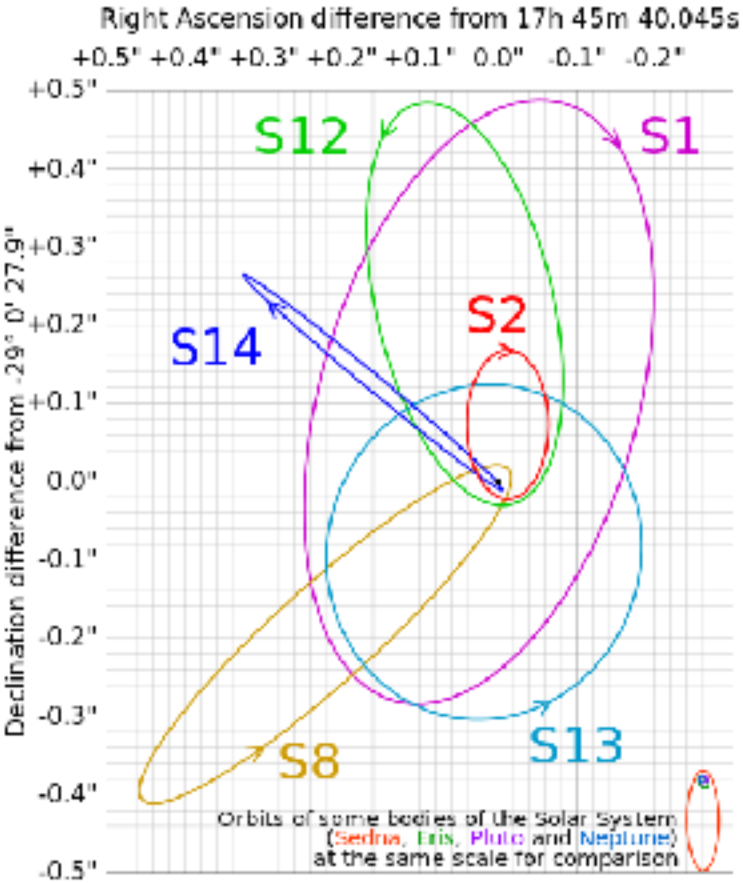


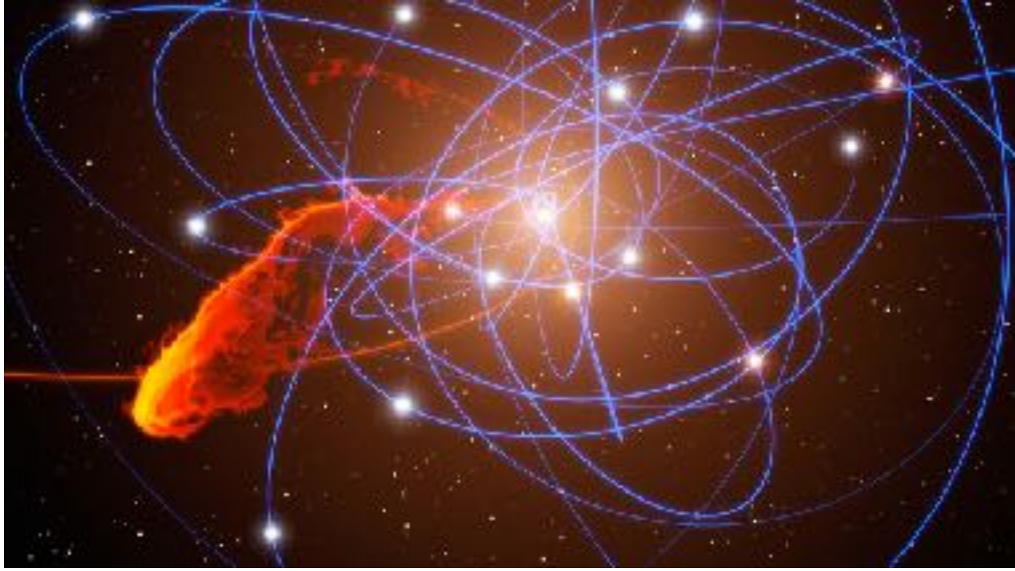


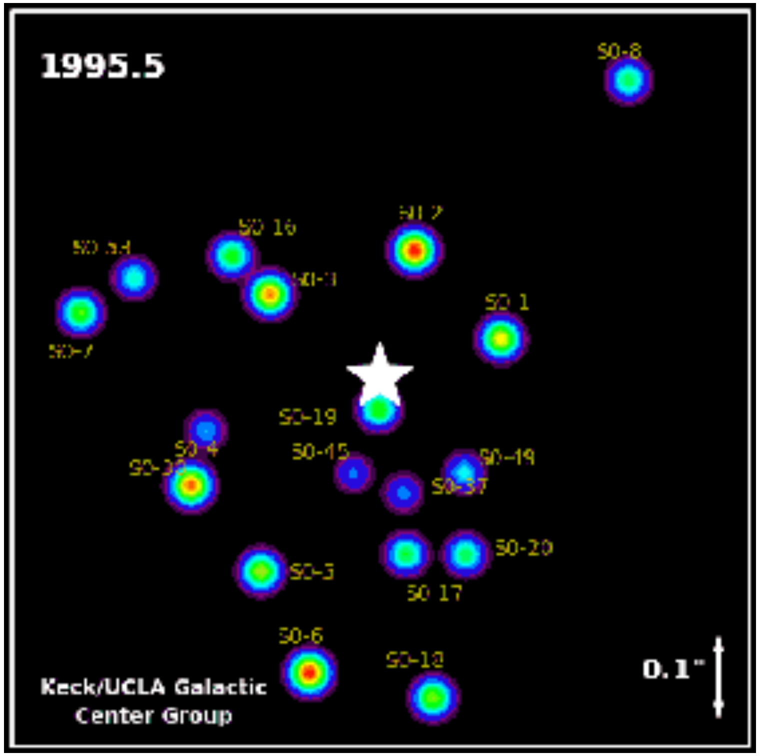
Inverse problem

1. Of the cause provided Newton's law
2. Of the model as it happened to be

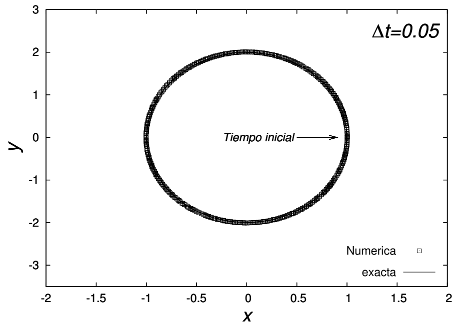
Example: Sgr A*

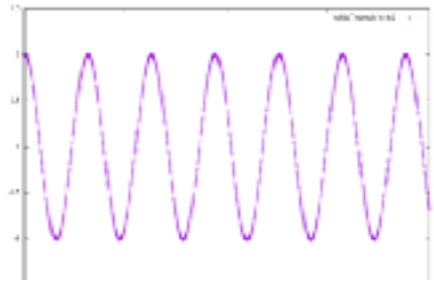




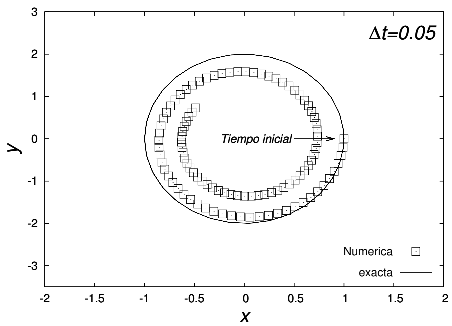


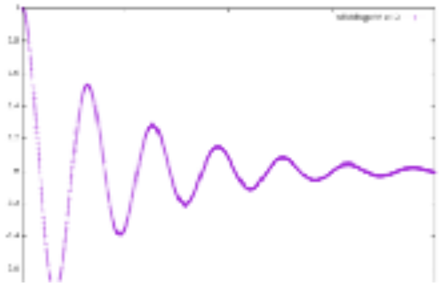
Finding a law and initial conditions: linear problems



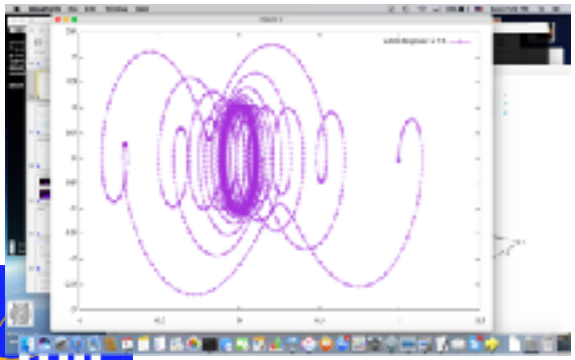


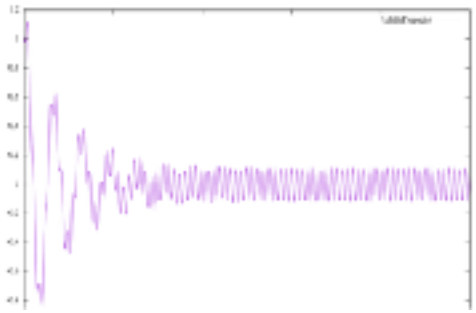
$$m\ddot{x} = -kx$$





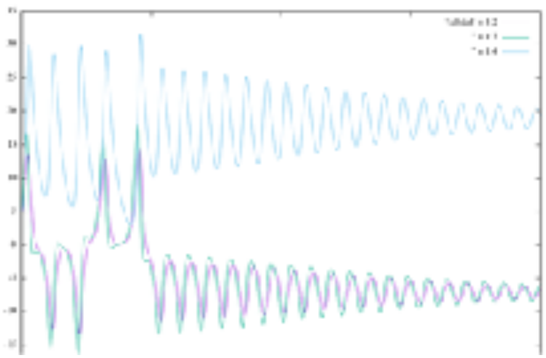
$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0.$$

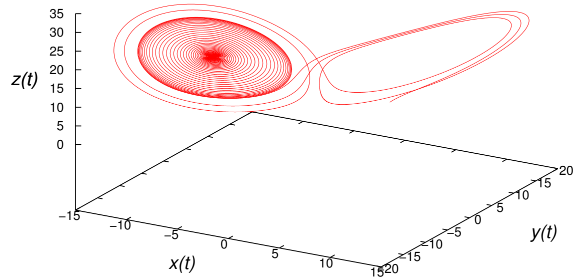




?

Solving Inverse problems depend A LOT on ***F*** (when there is one): see non-linear cases

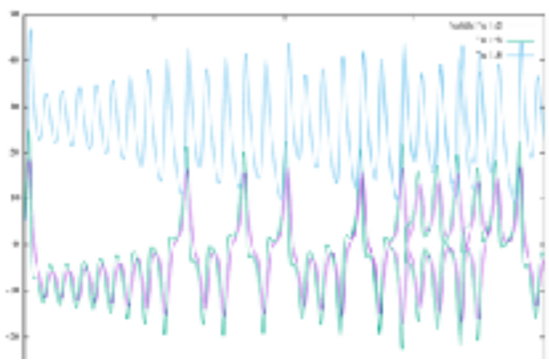


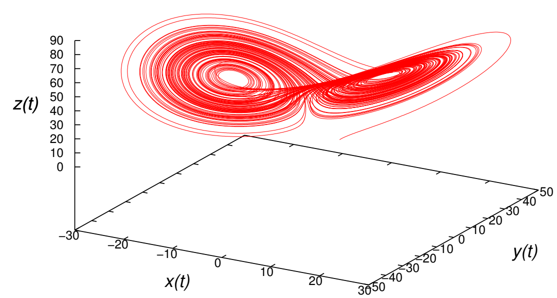


$$\frac{dx}{xt} = a(y - x)$$

$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$





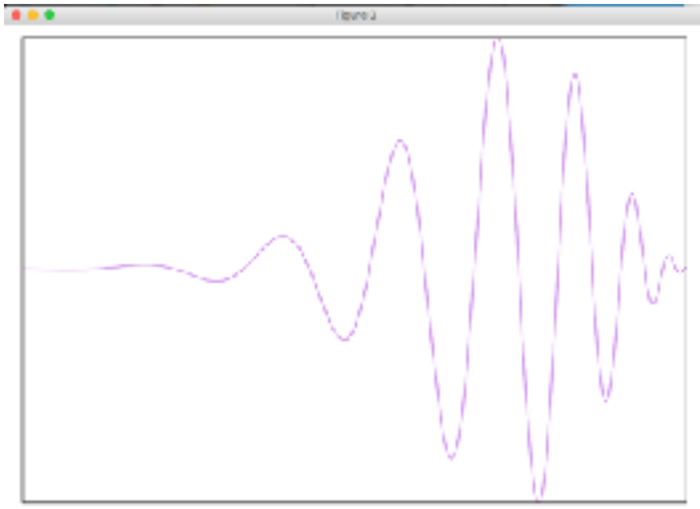
$a = 10$ $d_t = 0.001$

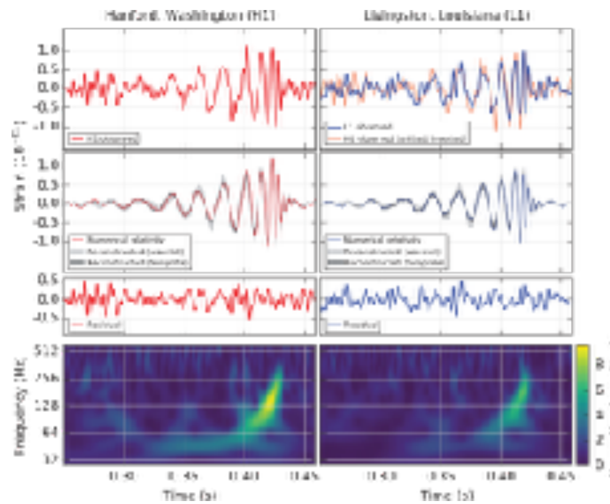
$b = 8/3$ $x_0 = y_0 = z_0 = 5.0$

$r = 10, 15, 50$ $0 \leq t \leq 100$

Imagine when the system is ruled by PDEs: computer power, time, time, time

Modeling data vs data science



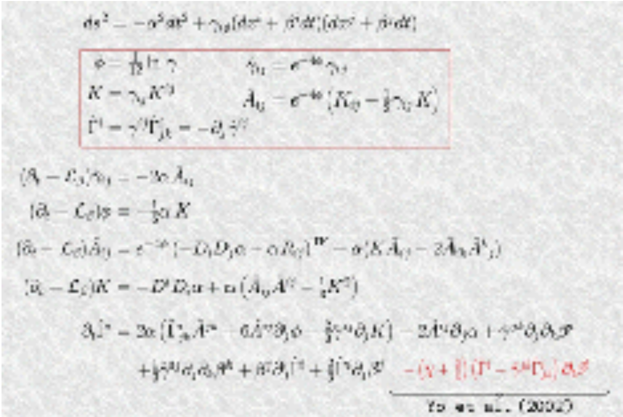



```
subroutine calcrhs(my_t,my_u)

  use numbers
  implicit none
  real(kind=8), intent(in) :: my_t
  real(kind=8), dimension(NE), intent(in) :: my_u

  rhs(1) = my_u(2)
  rhs(2) = - b / m * my_u(2) - k / m * my_u(1) + F0 / m * sin( CapOmega * my_t ) * exp(-(my_t-50.)**2/20.**2)

end subroutine calcrhs
```

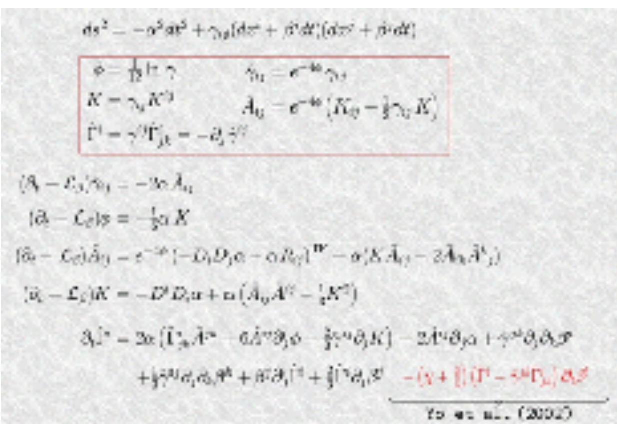


Solving Inverse problems depends A LOT on *F* (when there is one)
On the capacity to solve direct problems

And ***F*** is really challenging as one approaches a realistic model

Systems ruled by ODEs are actually friendly, one can still run millions of combinations of parameters and initial conditions

Then estimate one of the combinations with the minimum error



$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \nabla \cdot \left(\left(p + \frac{1}{2}\boldsymbol{B}^2 \right)\boldsymbol{I} + \rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} \right) = -(\nabla \cdot \boldsymbol{B})\boldsymbol{B} + \rho \boldsymbol{g},$$

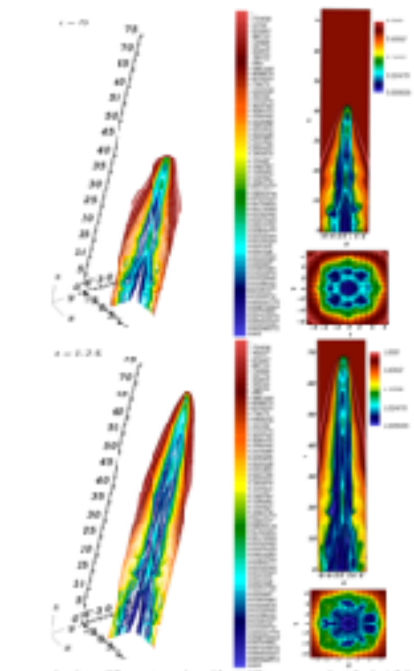
$$\frac{\partial E}{\partial t} + \nabla \cdot \left(\boldsymbol{v}\left(E + \frac{1}{2}\boldsymbol{B}^2 + p \right) - \boldsymbol{B}(\boldsymbol{B} \cdot \boldsymbol{v}) \right) = -\boldsymbol{B} \cdot (\nabla \psi) - \nabla \cdot ((\eta \cdot \boldsymbol{J}) \times \boldsymbol{B}) + \rho \boldsymbol{g} \cdot \boldsymbol{v},$$

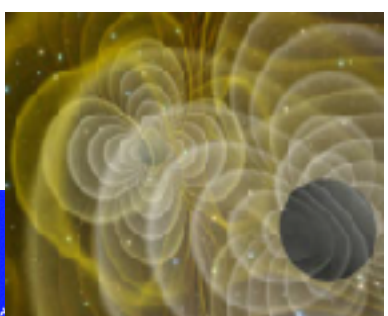
$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \cdot (\boldsymbol{B}\boldsymbol{v} - \boldsymbol{v}\boldsymbol{B} + \psi \boldsymbol{I}) = -\nabla \times (\eta \cdot \boldsymbol{J}),$$

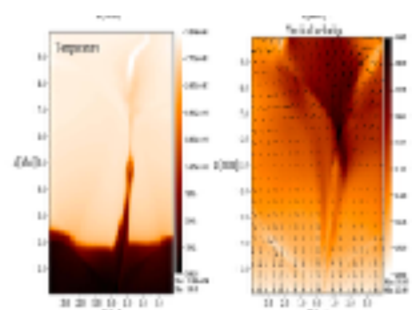
$$\frac{\partial \psi}{\partial t} + c_h^2 \nabla \cdot \boldsymbol{B} = -\frac{c_h^2}{c_p^2}\psi,$$

$$\boldsymbol{J} = \nabla \times \boldsymbol{B},$$

$$E = \frac{p}{(\gamma - 1)} + \frac{\rho v^2}{2} + \frac{\boldsymbol{B}^2}{2},$$





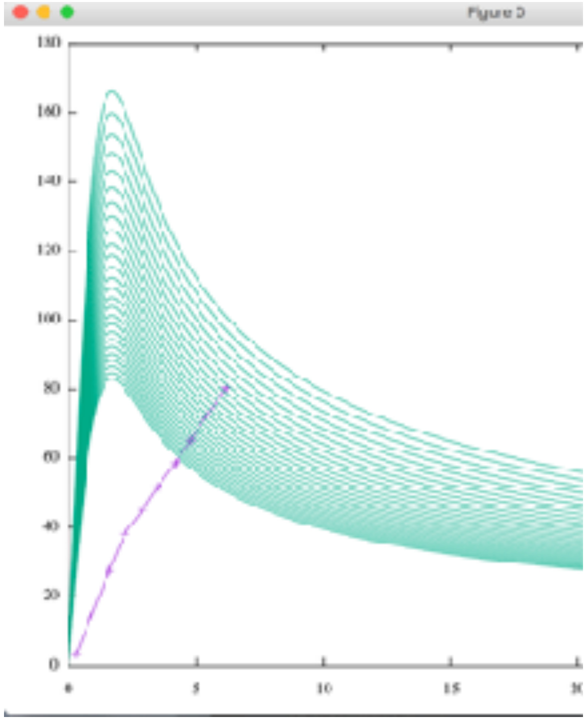


Methods of solution

Brutal force:

    Works fine for system ruled by ODEs (for instance)

    Does NOT work for complicated PDEs



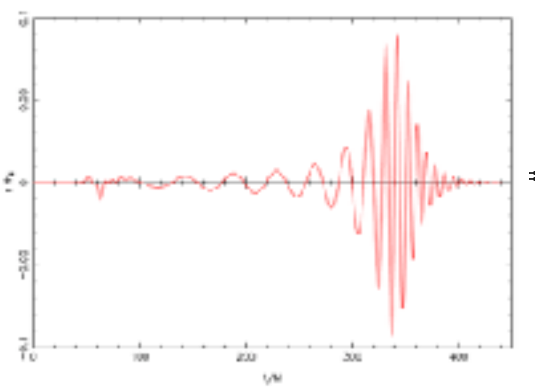
Genetic algorithms

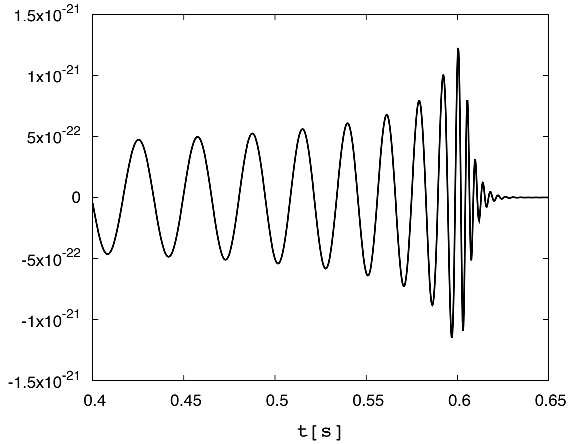
Artificial Intelligence methods

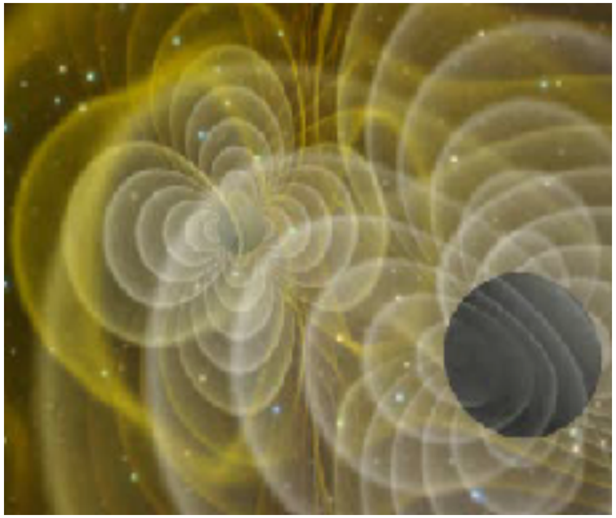
Inverse Problem 1: BBH … the direct problem first

In a simulation one estimates the Weyl Psi4 scalar

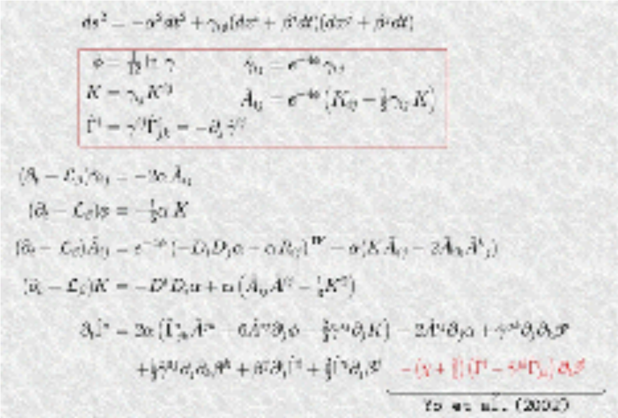
It is decomposed in spherical harmonics and get the dominant modes



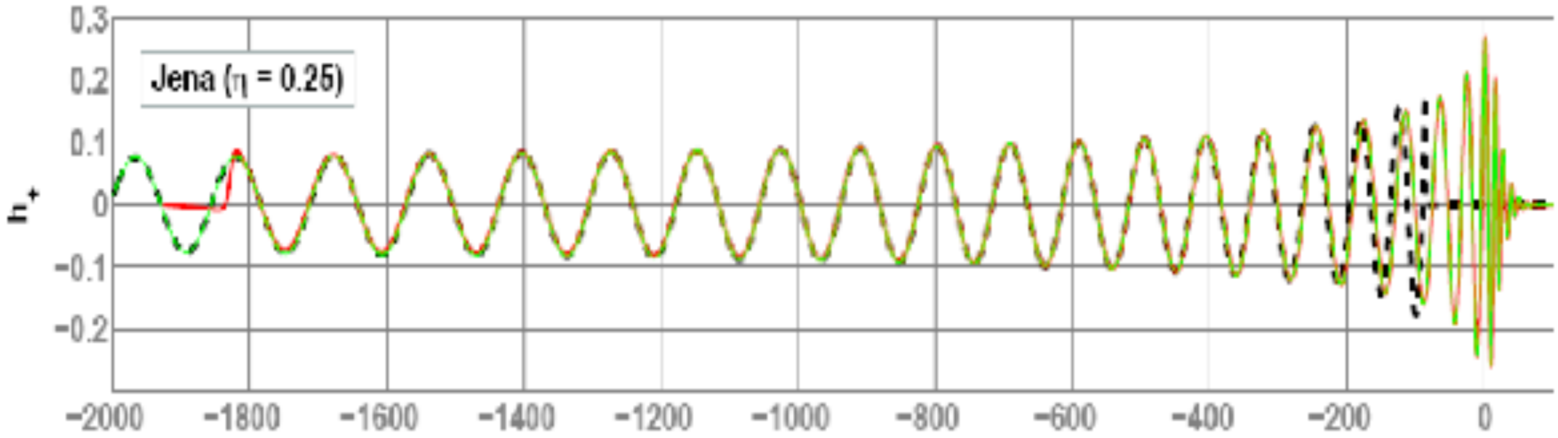


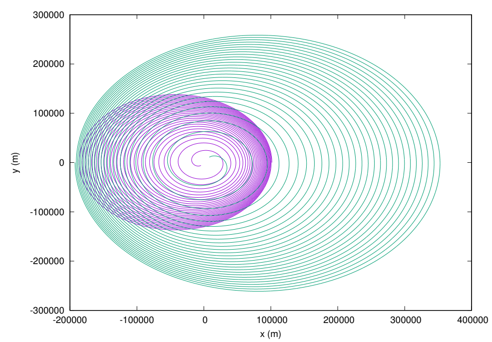


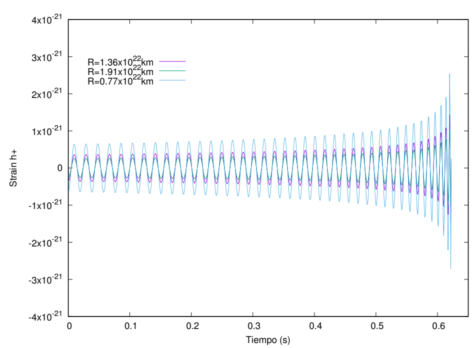
$$h = h_+ - ih_\times = \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' \Psi_4$$

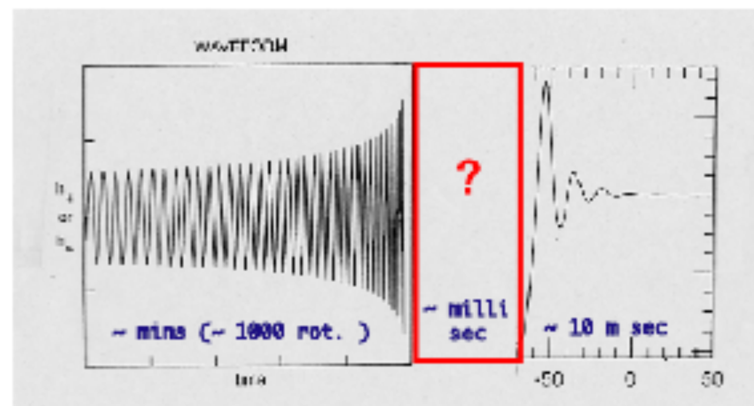


Inverse Problem 1: BBH … the direct problem first







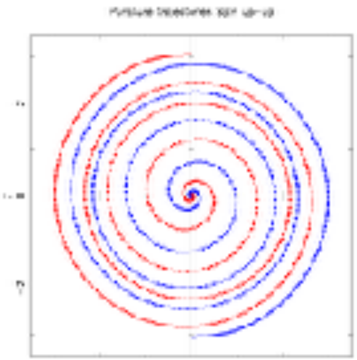


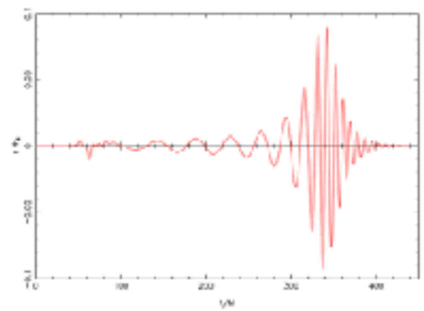
INSPIRAL → COALESCE → BLACKHOLE FORMATION

Innermost Stable Circular Orbit?

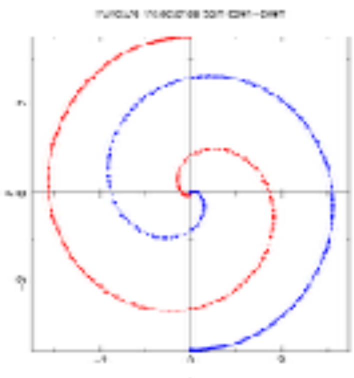
Post Newtonian Approx.
→ Numerical Relativity
→ BH. Perturbation

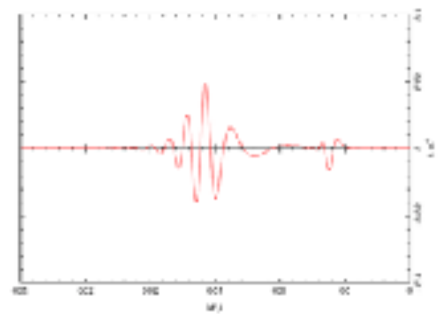
Direct problem: exploration a nightmare



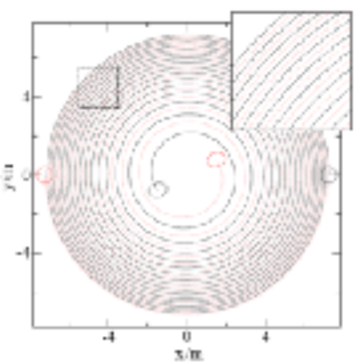


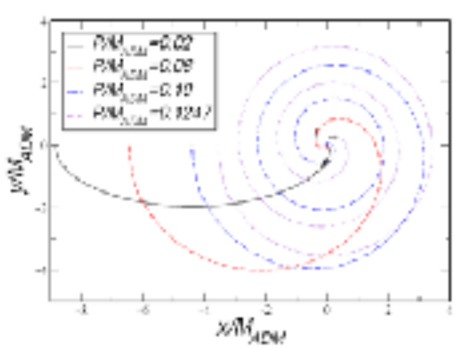
$\uparrow s_1 \; \uparrow L \; \uparrow s_2$

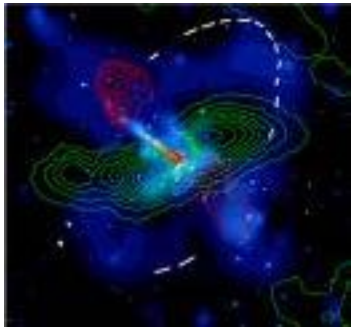




$\downarrow s_1 \; \uparrow L \; \downarrow s_2$







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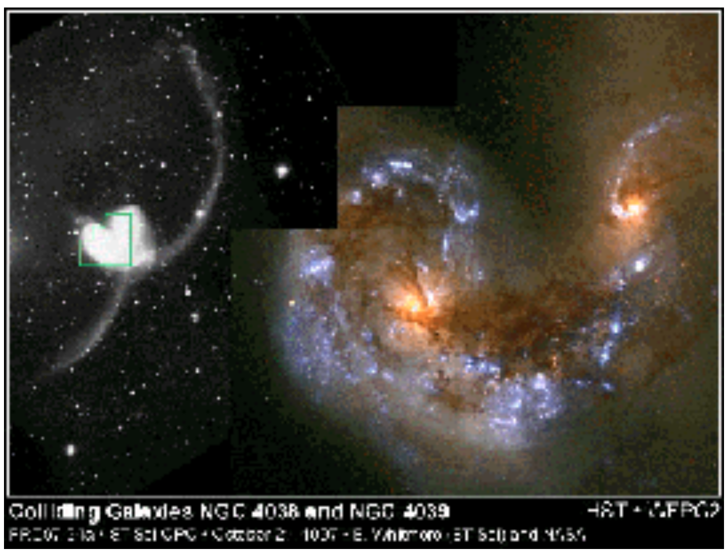
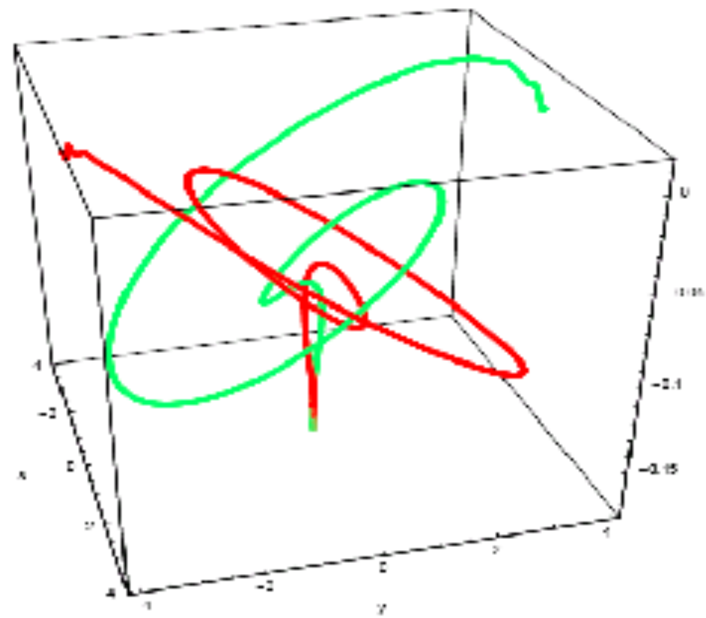
Direct problem: kicks

Escape velocities:

Globulares clusters 30Km/s
dSph 20-100km/2
dE 100-300km/s
giant galacies 1000km/2





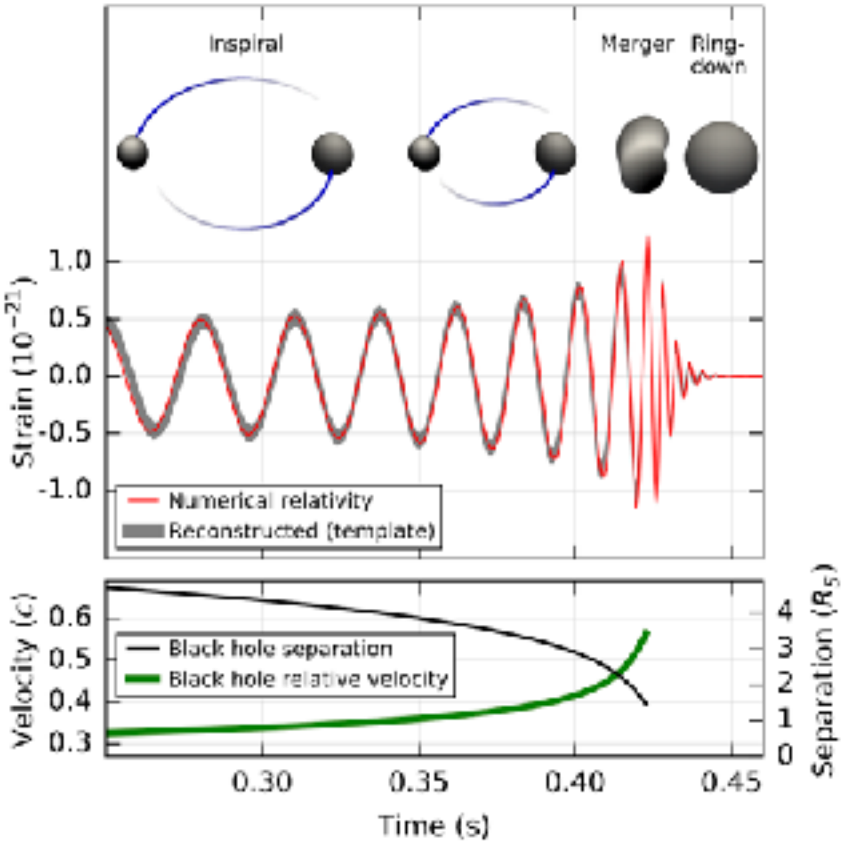
Numerical Relativity: TOO SLOW – too expensive

PostNewtonian approximation: *leading order*

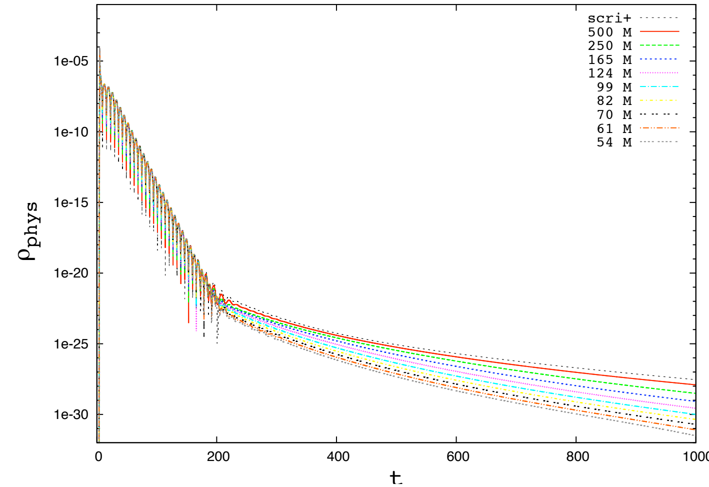
$$h^{insp}(t) = \frac{4GM\eta}{D_L c^2}\left(\frac{GM}{c^3}\frac{d\phi}{dt}\right)^{2/3}\cos[2\phi(t)]$$

D_L es la distancia a la fuente

$\eta = \mu / M$



Plus ringdown



Ringdown

$$h(t) = \sum_{n=0}^{N-1} A_n e^{-i\sigma_n (t - t_{match})}$$

n overtone

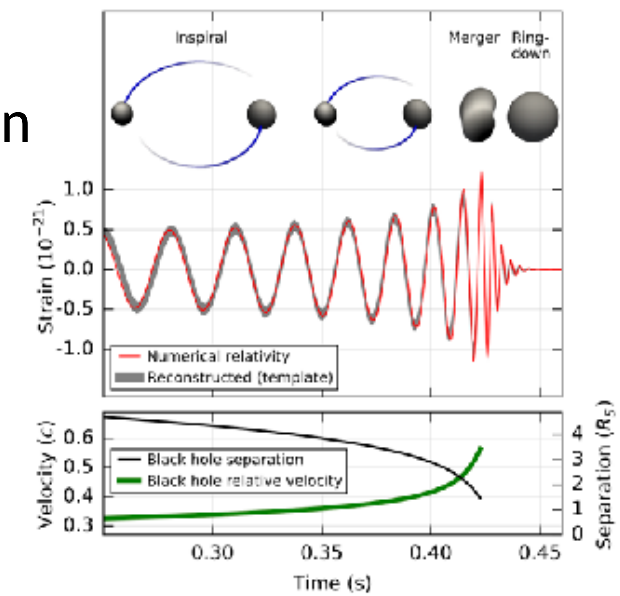
N number of overtones considered

A_n Amplitudes determining the MATCHING

$\sigma_n = \omega_n - i\alpha_n$

$\omega_n > 0$ son las frecuencias de oscilación

$\sigma_n > 0$ son el inverso del tiempo de decaimien

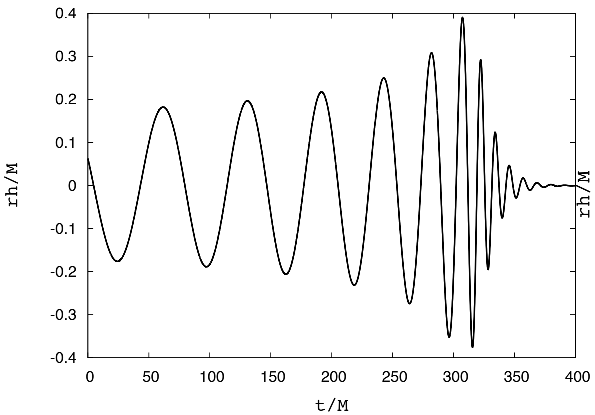


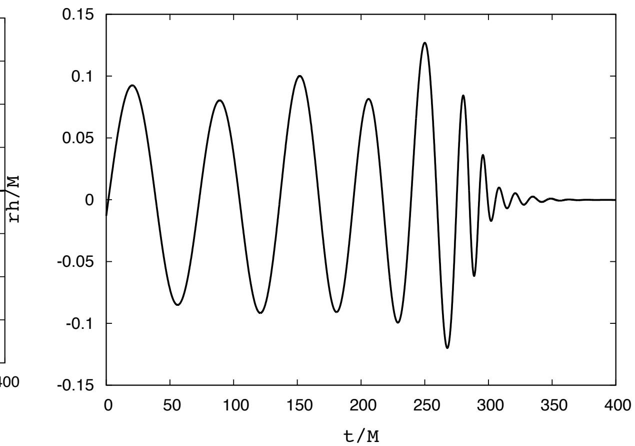
These quantities depend on the MASS

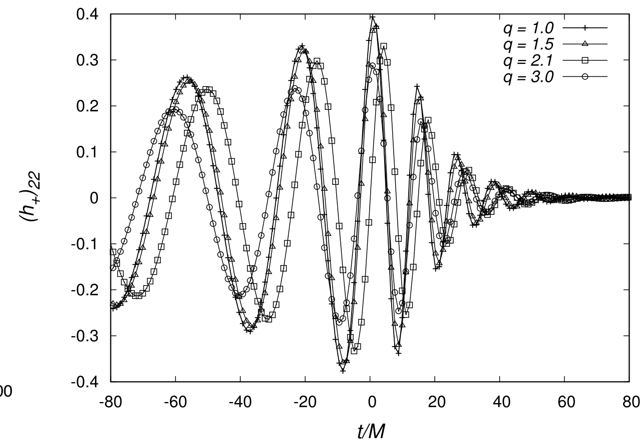
And SPIN of the ***FINAL BLACK HOLE***

Inverse Problem 1: BBH parameters from GW strain

$$M = m_1 + m_2, q = \frac{m_1}{m_2}, \chi_1 = \frac{S_1}{m_1^2}, \chi_2 = \frac{S_2}{m_2^2}$$





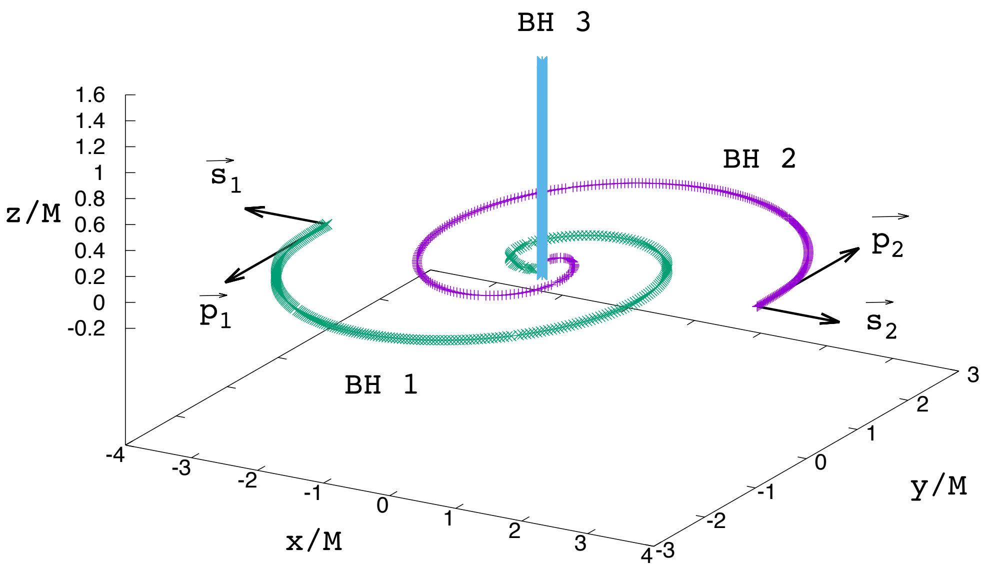


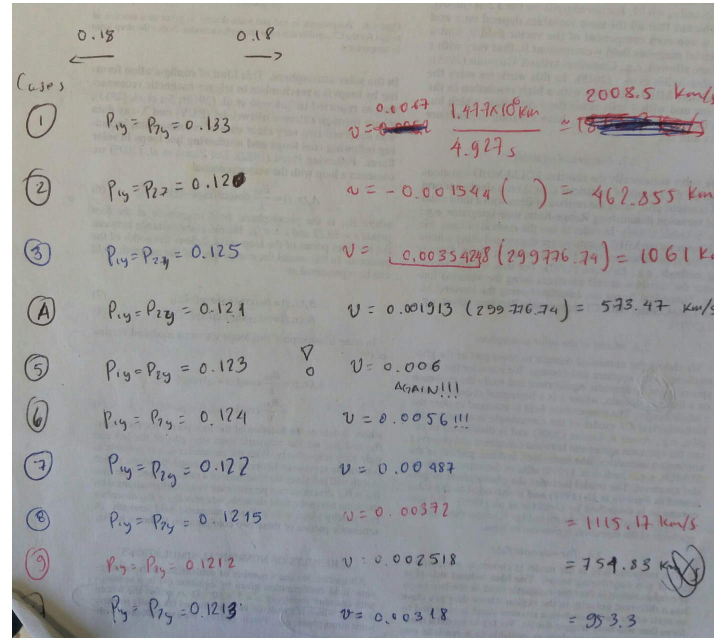
Parameter estimates in binary black hole collisions using neural networks
M. Carrillo, M. Gracia-Linares, J. A. González, F. S. Guzmán
Gen. Rel. Grav. 48, 141 (2016)

One parameter Binary Black Hole inverse problem using a sparse training set
M. Carrillo, M. Gracia-Linares, J. A Gonzáles, F S Guzmán
Int. J. Mod. Phys. D 27, 1850043 (2018)

Inverse Problem 2: kicked BH inverse problem

Of the cause
Of the cause + model





$$
\begin{aligned}
\vec{V}_{\text{recoil}}(q,\vec{\alpha}) &= v_m\,\hat{e}_1 + v_\perp(\cos\xi\,\hat{e}_1 + \sin\xi\,\hat{e}_2) + v_\parallel\,\hat{n}_\parallel,\\
v_m &= A\frac{\eta^2(1-q)}{(1+q)}\left[1+B\,\eta\right],\\
v_\perp &= H\frac{\eta^2}{(1+q)}\left[(1+B_H\,\eta)\,(\alpha_2^\parallel - q\alpha_1^\parallel) + H_S\frac{(1-q)}{(1+q)^2}\,(\alpha_2^\parallel + q^2\alpha_1^\parallel)\right],\\
v_\parallel &= K\frac{\eta^2}{(1+q)}\Big[(1+B_K\,\eta)\left|\alpha_2^\perp - q\alpha_1^\perp\right|\cos(\Theta_\Delta-\Theta_0)\\
&\quad + K_S\frac{(1-q)}{(1+q)^2}\left|\alpha_2^\perp + q^2\alpha_1^\perp\right|\cos(\Theta_S-\Theta_1)\Big],
\end{aligned}
$$

Remnant Masses, Spins and Recoils from the Merger of Generic Black-Hole Binaries
Carlos O. Lousto, Manuela Campanelli, Yosef Zlochower, Hiroyuki Nakano (RIT)
Comments: 15 pages, 2 figures. Rewritten and added new section.
Journal-ref: Class.Quant.Grav.27:114006,2010

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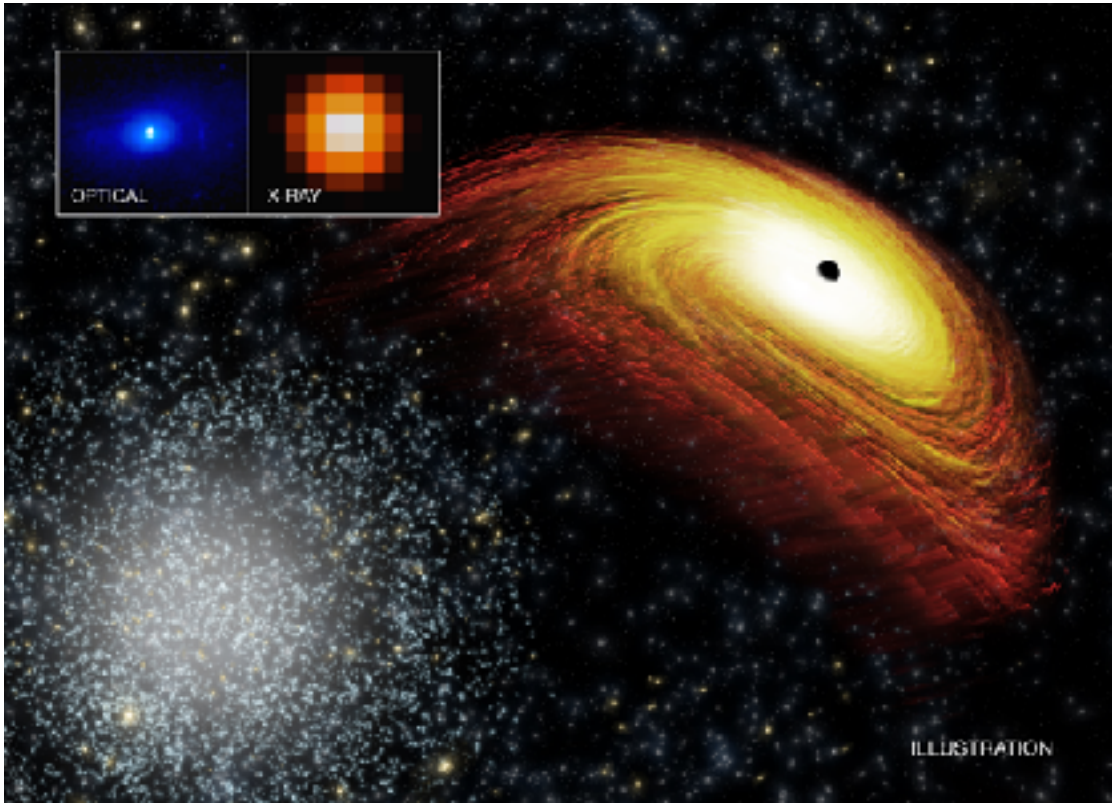
Inverse Problem 3: QSO 3C 186 (radio-loud quasar)

Best candidate for GW recoil kicked black hole



11kpc off-set from center … Broad emission lines -> -2140 \pm 390km/s

Bleeding edge

MODELING THE BLACK HOLE MERGER OF QSO 3C 186

Carlos O.Lousto, Yosef Zlochower, and Manuela Campanelli
Center for Computational Relativity and Gravitation,
and School of Mathematical Sciences, Rochester Institute of Technology, 85 Lomb Memorial Drive, Rochester, New York 14623
Draft version April 5, 2017

ABSTRACT

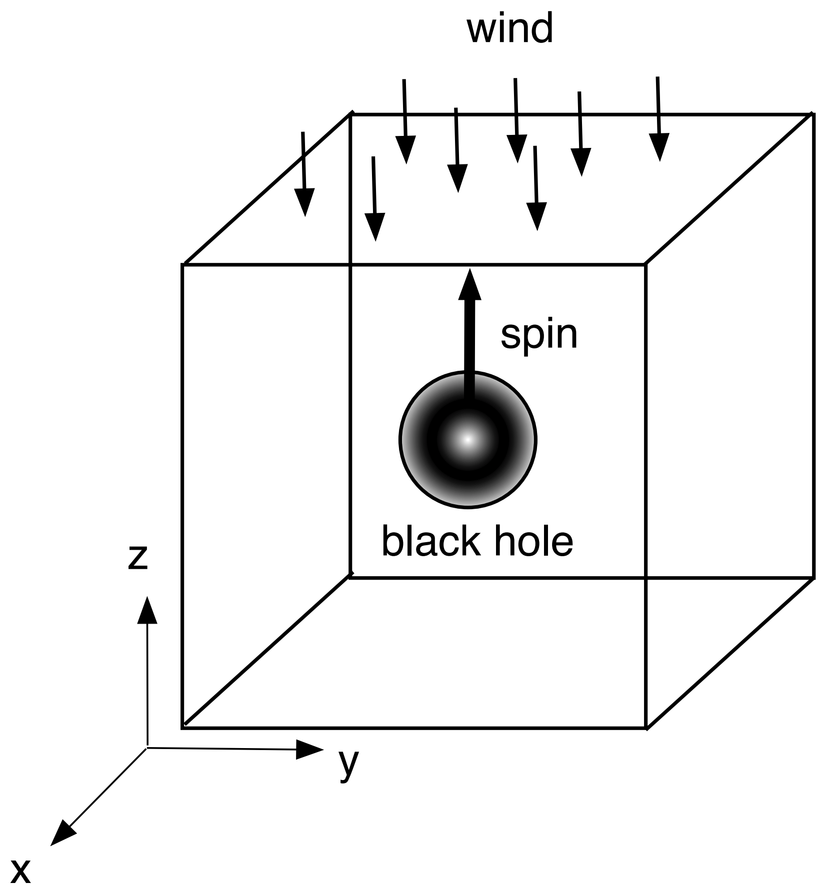
Recent detailed observations of the radio-loud quasar 3C 186 indicate the possibility that a supermassive recoiling black hole is moving away from the host galaxy at a speed of nearly 2100km/s. If this is the case, we can model the mass ratio and spins of the progenitor binary black hole using the results of numerical relativity simulations. We find that the black holes in the progenitor must have comparable masses with a mass ratio $q = m_1/m_2 > 1/4$ and the spin of the primary black hole must be $\alpha_2 = S_2/m_2^2 > 0.4$. The final remnant of the merger is bounded by $\alpha_f > 0.45$ and at least 4% of the total mass of the binary system is radiated into gravitational waves. We consider four different pre-merger scenarios that further narrow those values. Assuming, for instance, a cold accretion driven merger model, we find that the binary had comparable masses with $q = 0.70^{+0.29}_{-0.21}$ and the normalized spins of the larger and smaller black holes were $\alpha_2 = 0.94^{+0.06}_{-0.22}$ and $\alpha_1 = 0.95^{+0.05}_{-0.09}$. We can also estimate the final recoiling black hole spin $\alpha_f = 0.93^{+0.02}_{-0.03}$ and that the system radiated $9.6^{+0.8}_{-1.4}\%$ of its total mass, making the merger of those black holes the most energetic event ever observed.
Keywords: supermassive black holes — binary merger — gravitational recoils

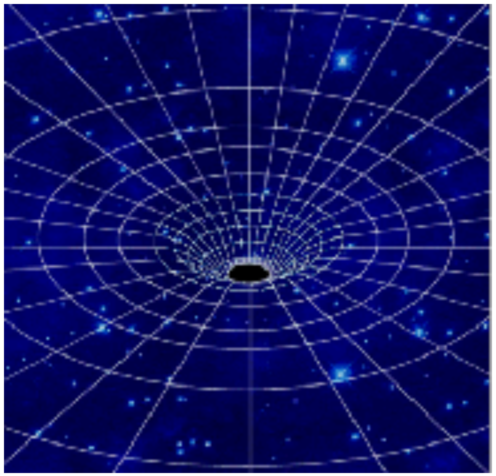
Direct problem

$$T^{\mu\nu} = \rho_0 h u^\mu u^\nu + p g^{\mu\nu}$$

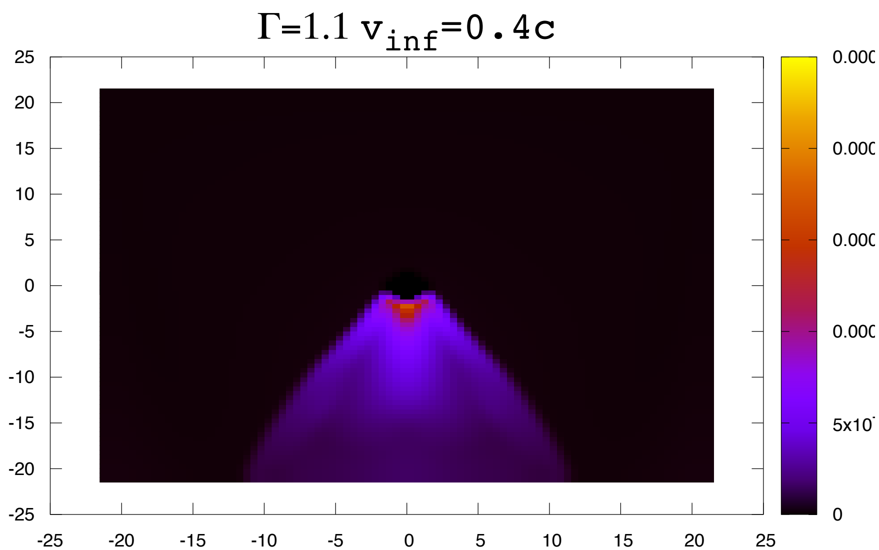
$$\frac{\partial u}{\partial t} + \frac{\partial F^i(u)}{\partial x^i} = S$$

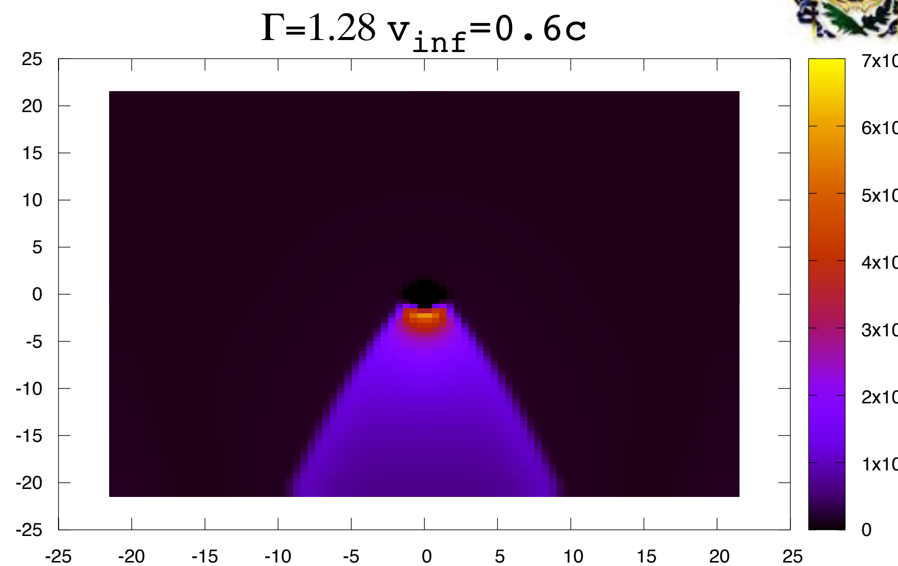
$$u = \begin{bmatrix} D \\ J_1 \\ J_2 \\ J_3 \\ \tau \end{bmatrix}, F^i = \begin{bmatrix} \alpha\left(v^i - \frac{\beta^i}{\alpha}\right)D \\ \alpha\left(v^i - \frac{\beta^i}{\alpha}\right)J_1 + \alpha\sqrt{\gamma}p\delta^i{}_1 \\ \alpha\left(v^i - \frac{\beta^i}{\alpha}\right)J_2 + \alpha\sqrt{\gamma}p\delta^i{}_2 \\ \alpha\left(v^i - \frac{\beta^i}{\alpha}\right)J_3 + \alpha\sqrt{\gamma}p\delta^i{}_3 \\ \alpha\left(v^i - \frac{\beta^i}{\alpha}\right) + \alpha\sqrt{\gamma}v^i p \end{bmatrix}, S = \begin{bmatrix} 0 \\ \alpha\sqrt{\gamma}T^{\mu\nu}g_{\nu\sigma}\Gamma^\sigma{}_{\mu 1} \\ \alpha\sqrt{\gamma}T^{\mu\nu}g_{\nu\sigma}\Gamma^\sigma{}_{\mu 2} \\ \alpha\sqrt{\gamma}T^{\mu\nu}g_{\nu\sigma}\Gamma^\sigma{}_{\mu 3} \\ \alpha\sqrt{\gamma}(T^{\mu 0}\partial_\mu \alpha - \alpha T^{\mu\nu}\Gamma^0{}_{\mu\nu}) \end{bmatrix}$$

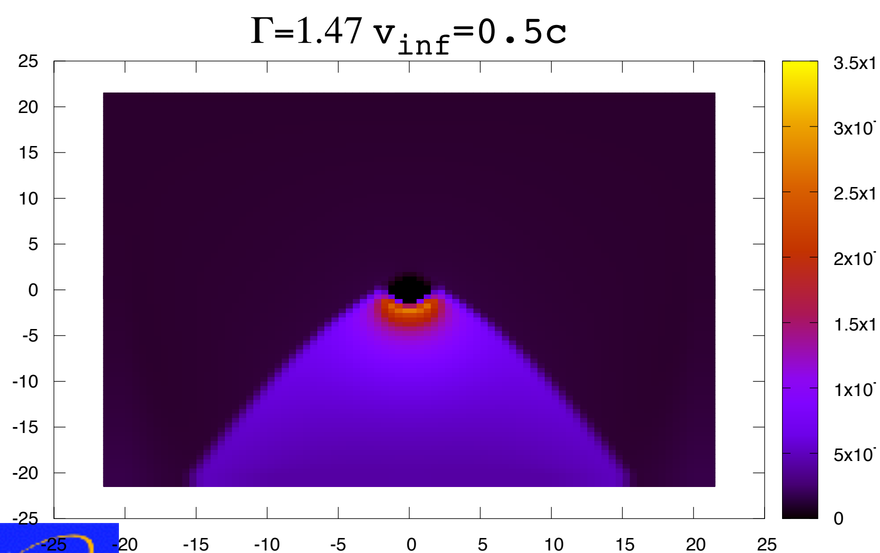


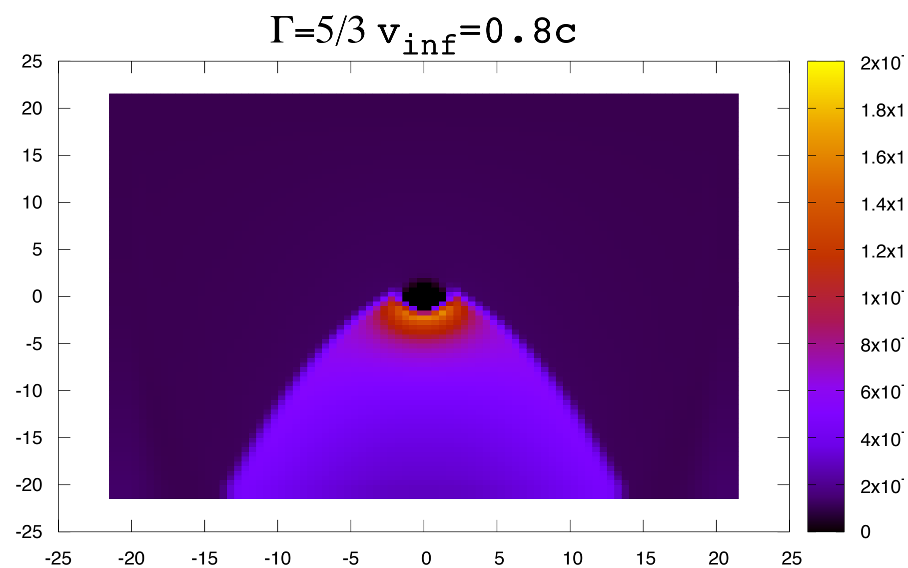


A sample: 4 out of 900

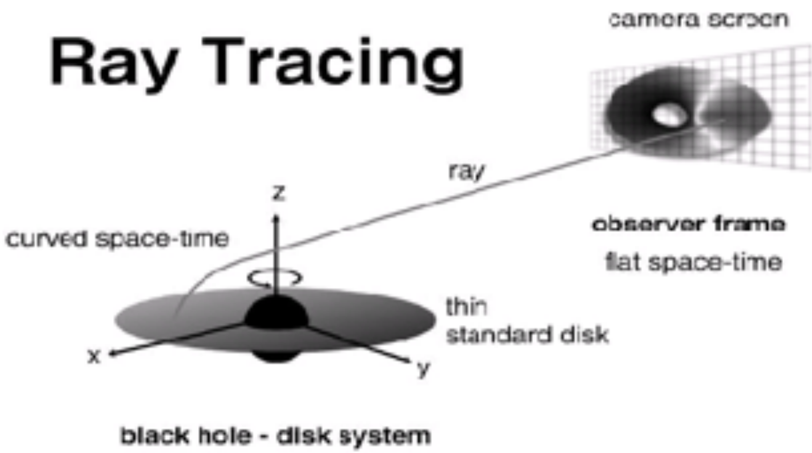


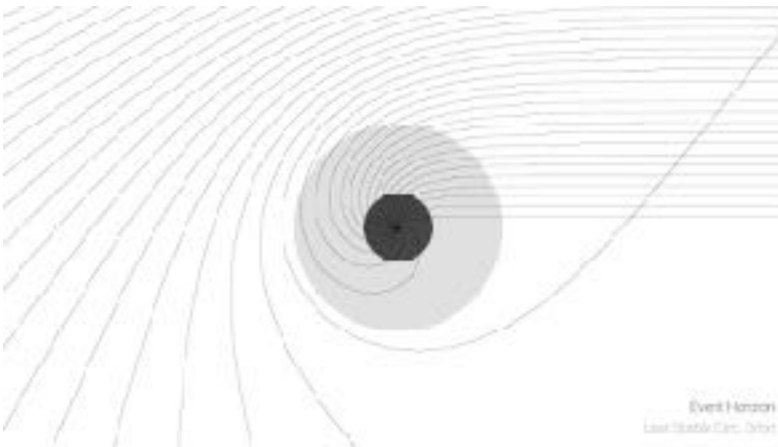




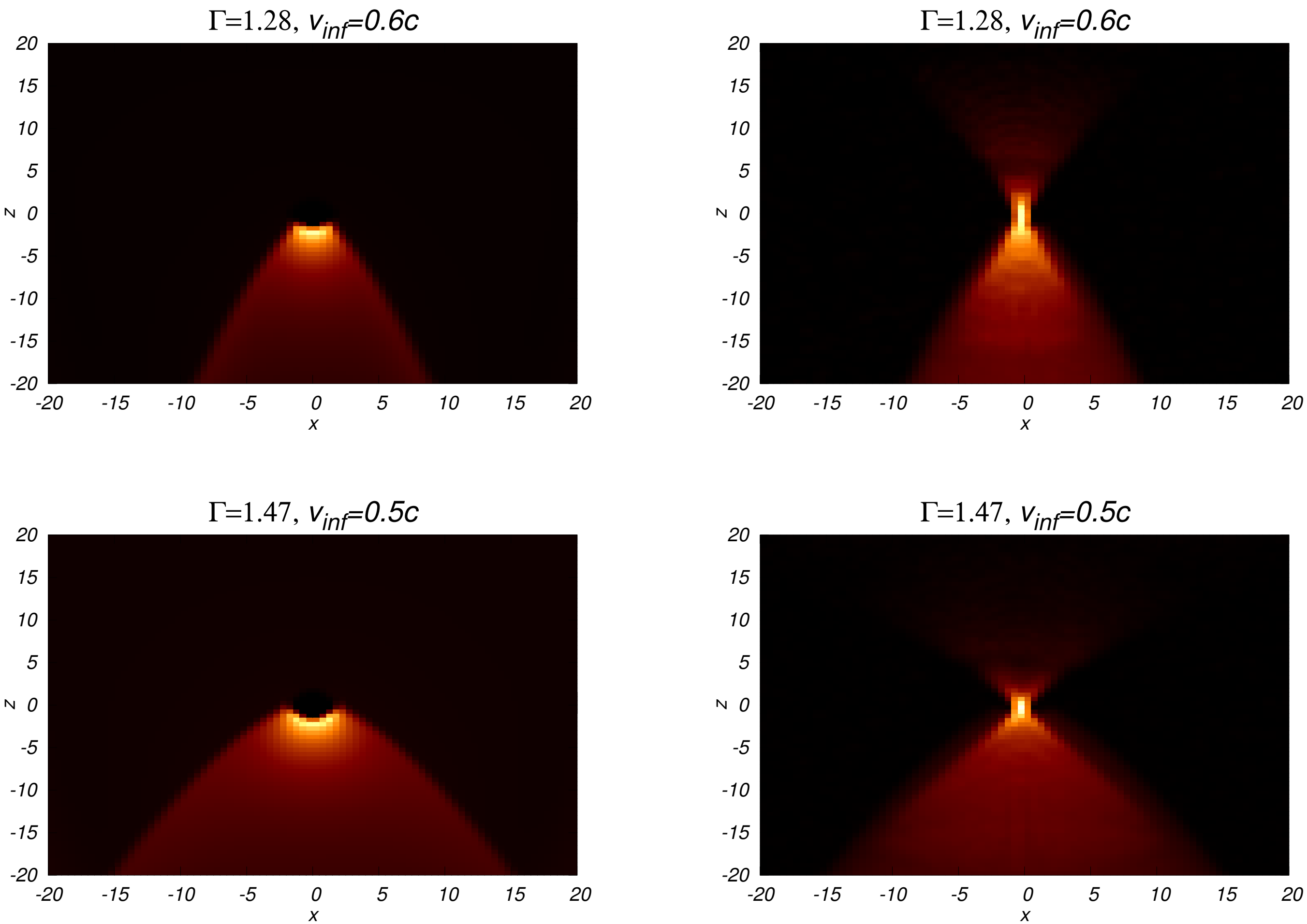


Ray tracing process



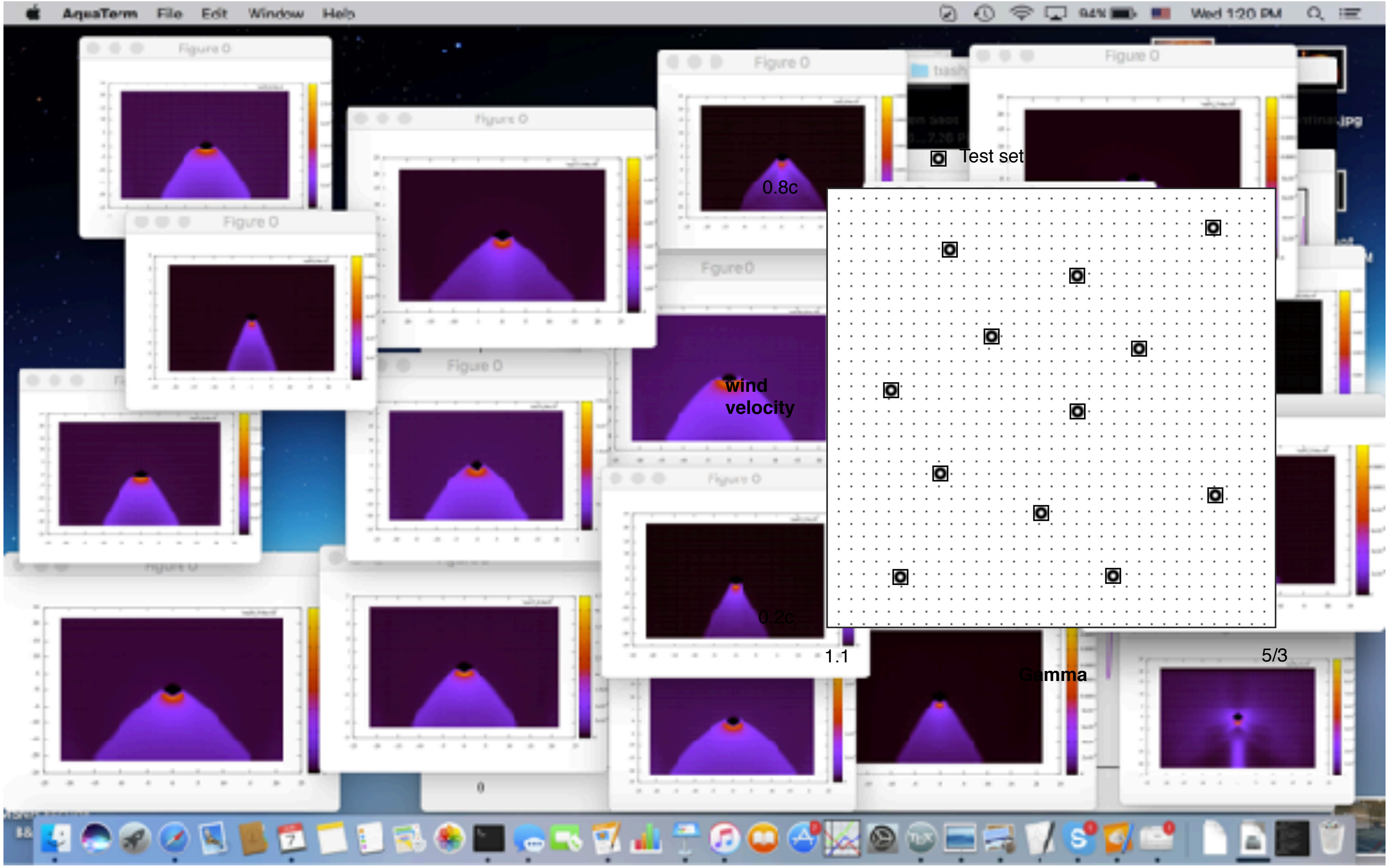


A little sample 2 out of 900



Characterizing the velocity of a wandering black hole and properties of the surrounding medium using convolutional neural networks
J. A González, F. S. Guzmán
Phys. Rev. D 97, 063001 (2018)

We prepared a sample of 900 of these runs

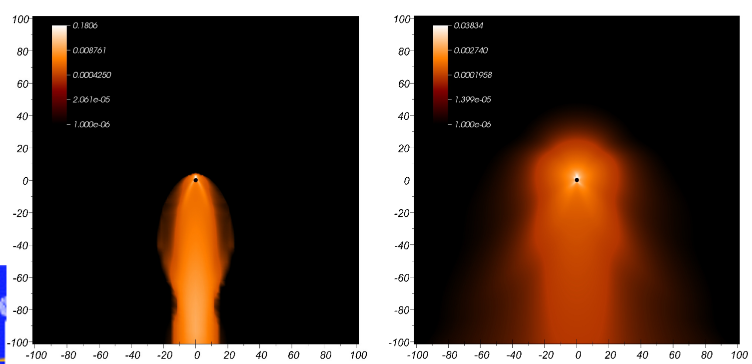


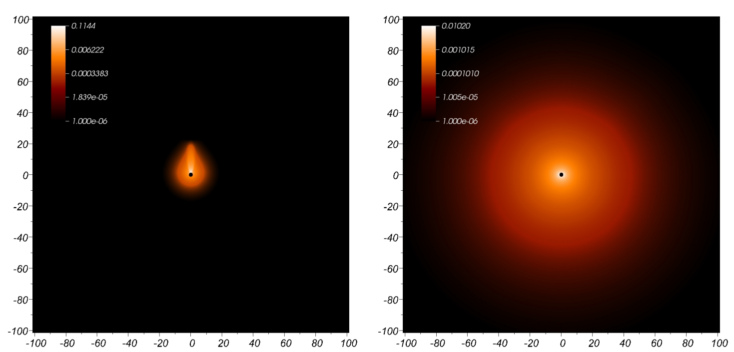
GOAL

Given an image track the following:

- Properties of the black hole candidates (spin, mass)
- Properties of the matter around (which are model dependent)
- These include equation of state -> degree of ionization
- Temperature - feedbacks the scattering properties
- Opacities (both thermal and scattering)
- Magnetic fields

QSO 3C 186





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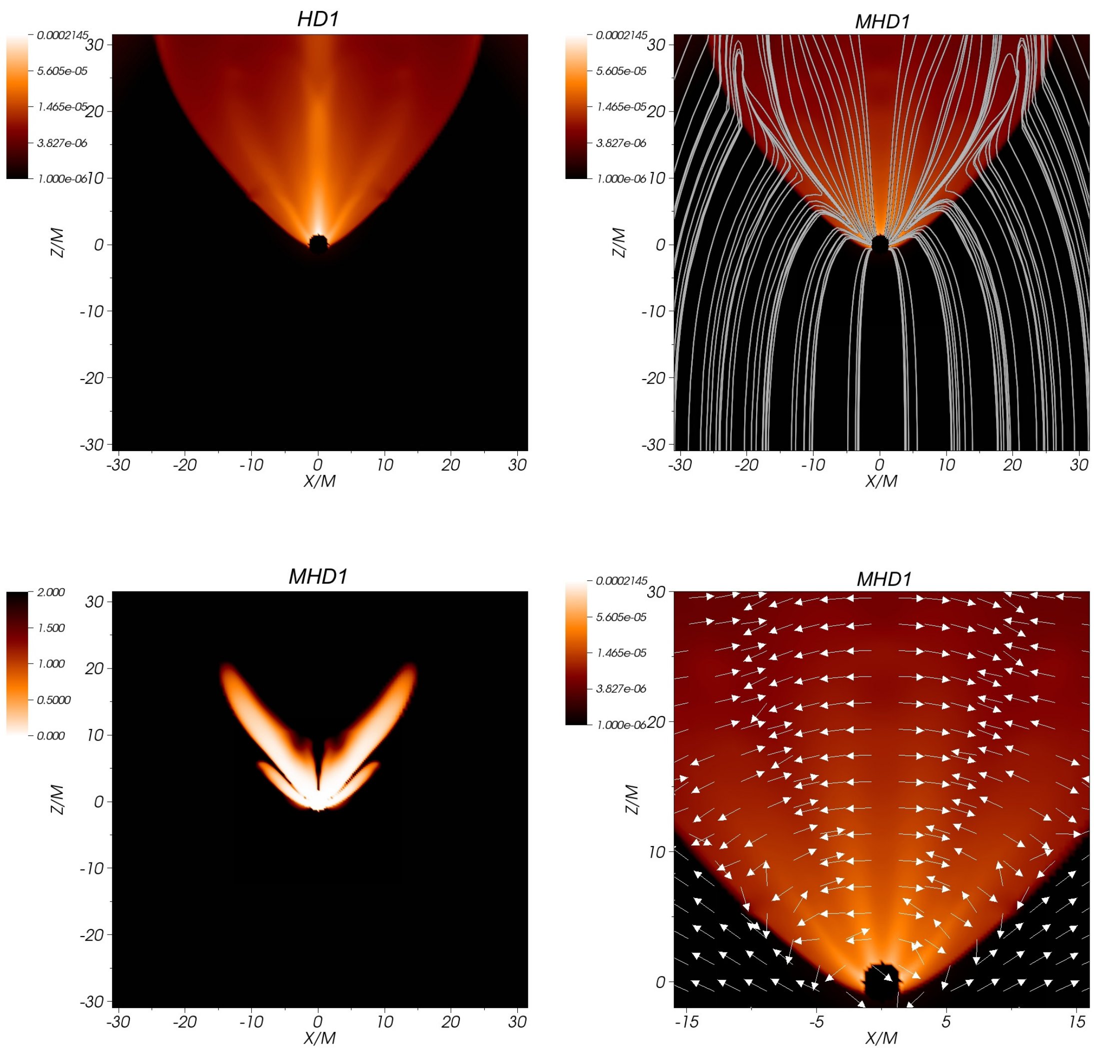
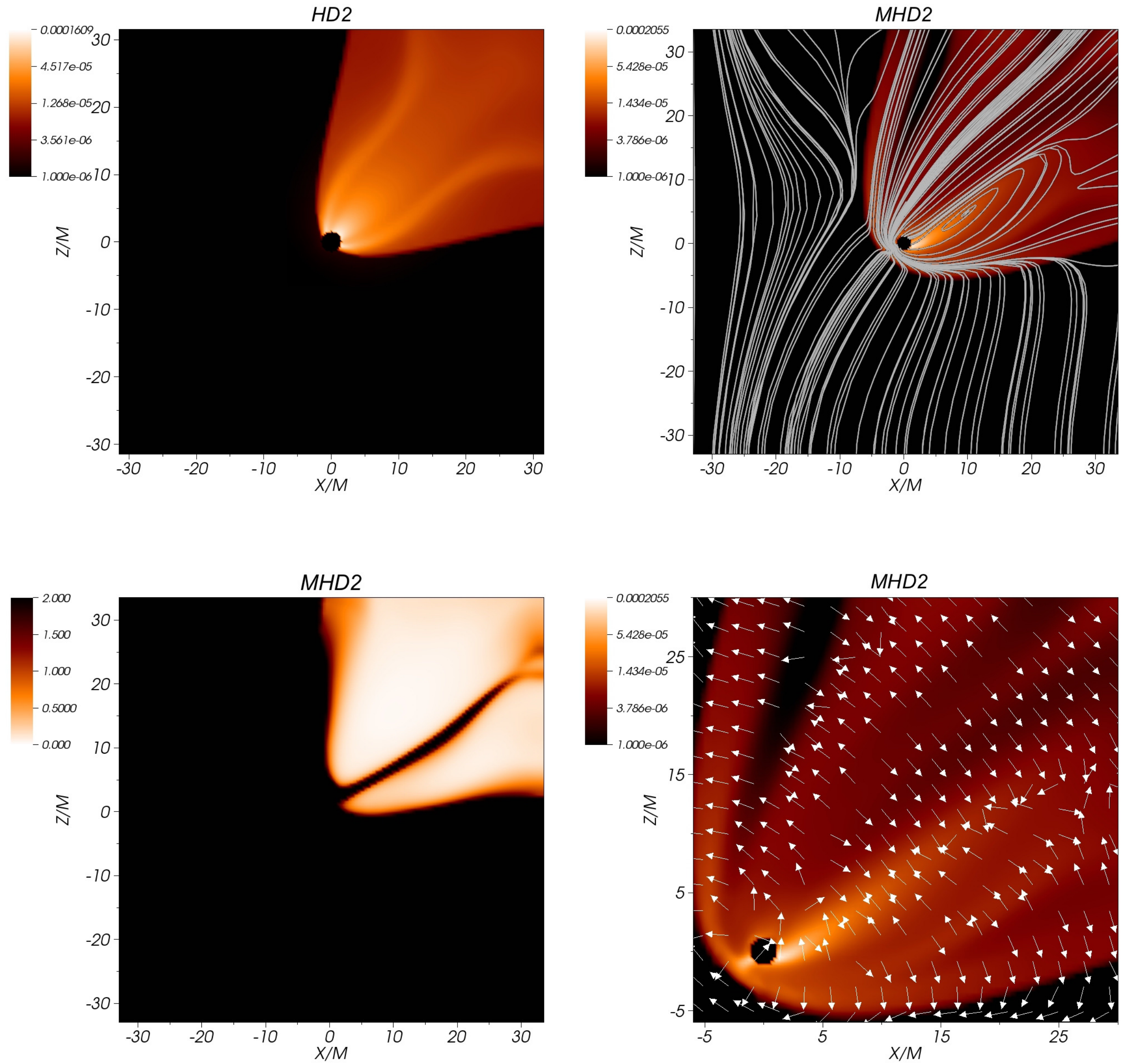
Magnetized winds

$$T^{\mu\nu} = (\rho h + b^2) u^\mu u^\nu + \left(p + \frac{b^2}{2} \right) g^{\mu\nu} - b^\mu b^\nu$$

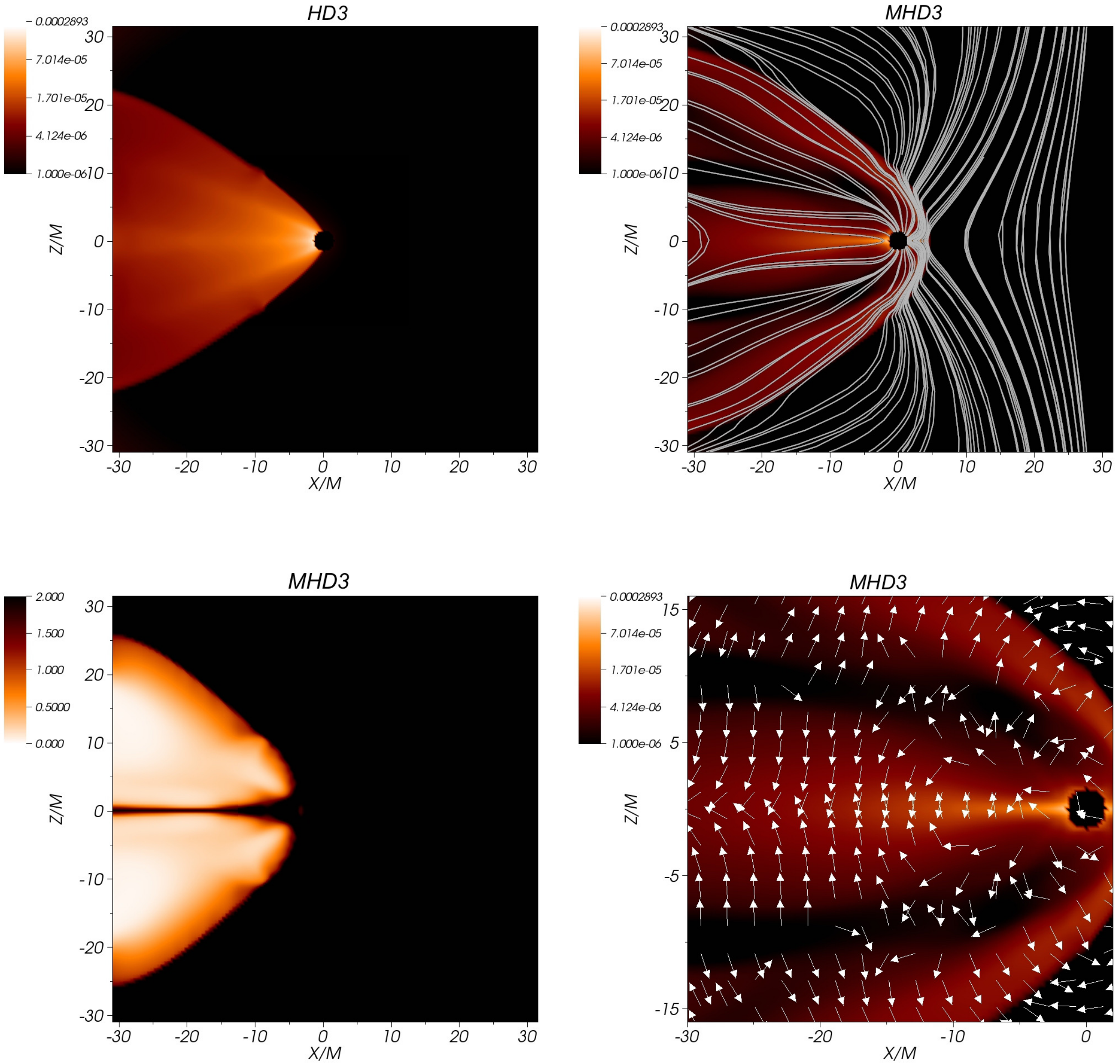


MHD

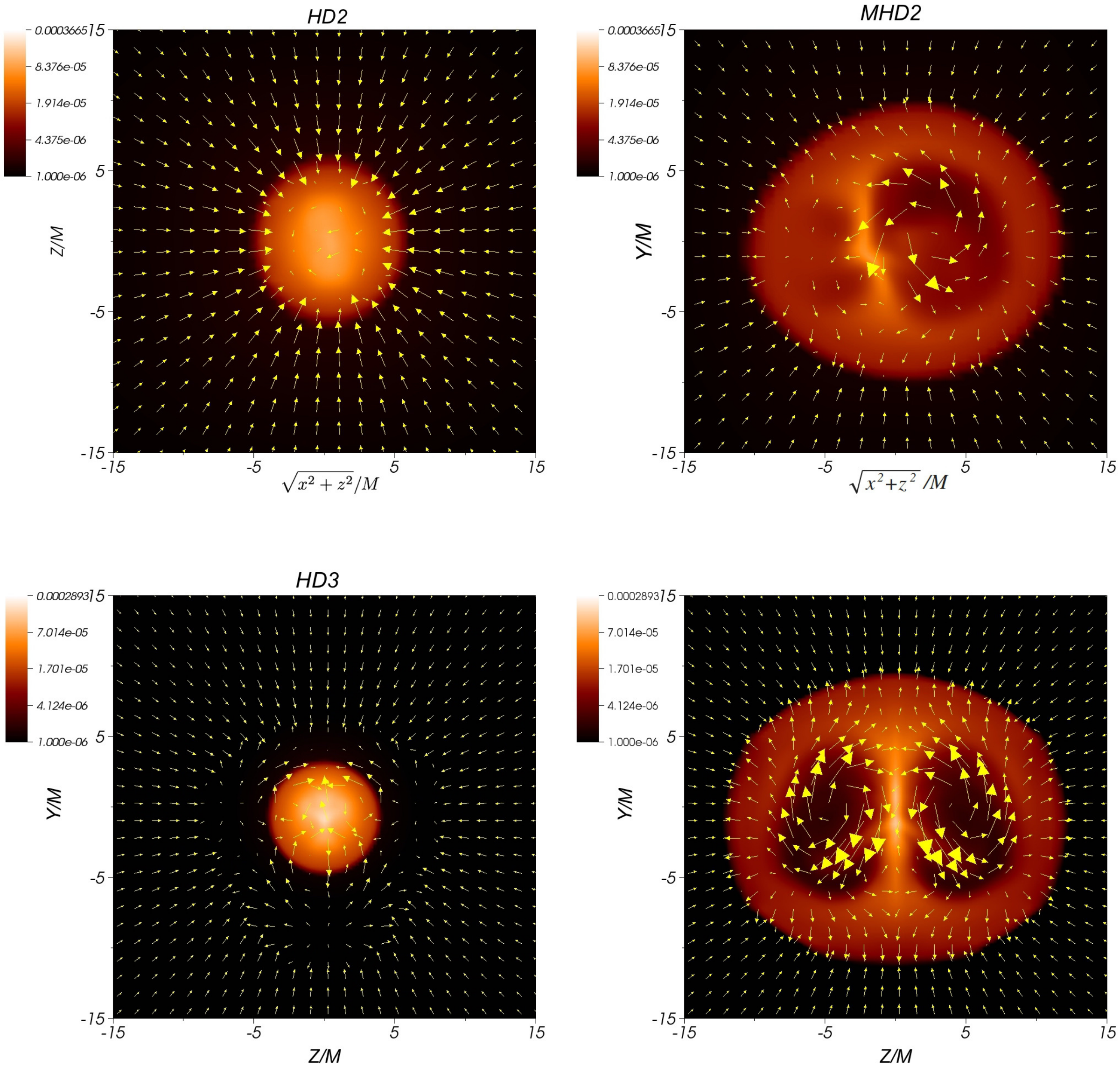


HD2
MHD2
MHD2
MHD2
0.0001609
4.517e-05
1.268e-05
3.561e-06
1.000e-06
0.0002055
5.428e-05
1.434e-05
3.786e-06
1.000e-06
2.000
1.500
1.000
0.5000
0.000
X/M
Z/M

MHD


HD3
MHD3
MHD3
MHD3
0.0002893
7.014e-05
1.701e-05
4.124e-06
1.000e-06
2.000
1.500
1.000
0.5000
0.000
Z/M
X/M
30
20
10
0
-10
-20
-30
15
5
-5
-15

MHD



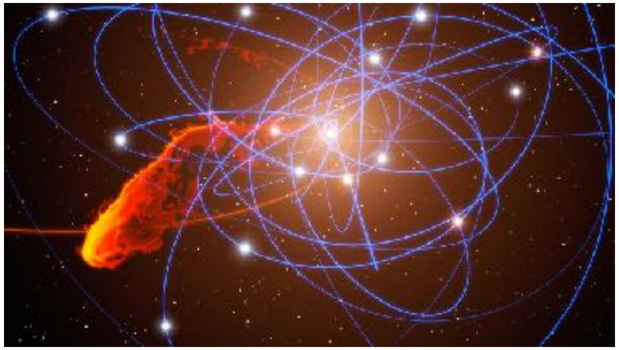
HD2
MHD2
HD3
0.0003665
8.376e-05
1.914e-05
4.375e-06
1.000e-06
0.0002893
7.014e-05
1.701e-05
4.124e-06
1.000e-06
Z/M
Y/M
$\sqrt{x^2+z^2}/M$
Z/M
15
5
-5
-15

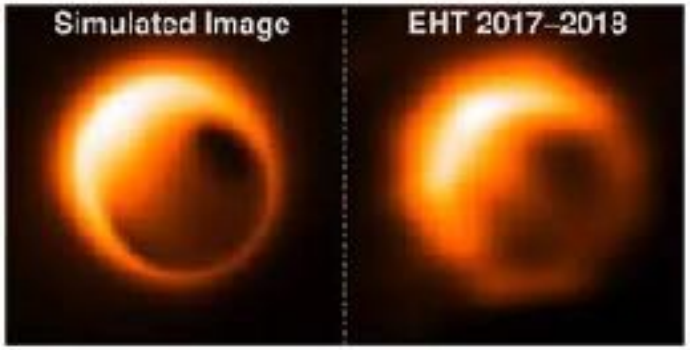
Inverse Problem 4: EHT Inverse problem

Spin and orientation — M is estimated from stars around

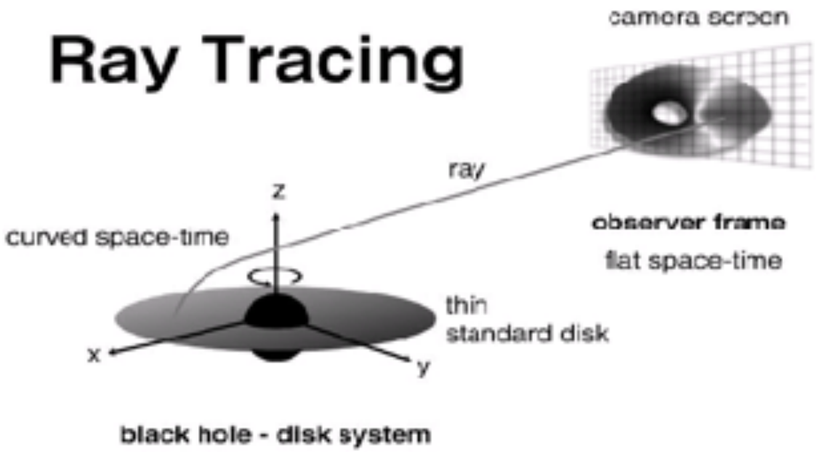
Is this a black hole?

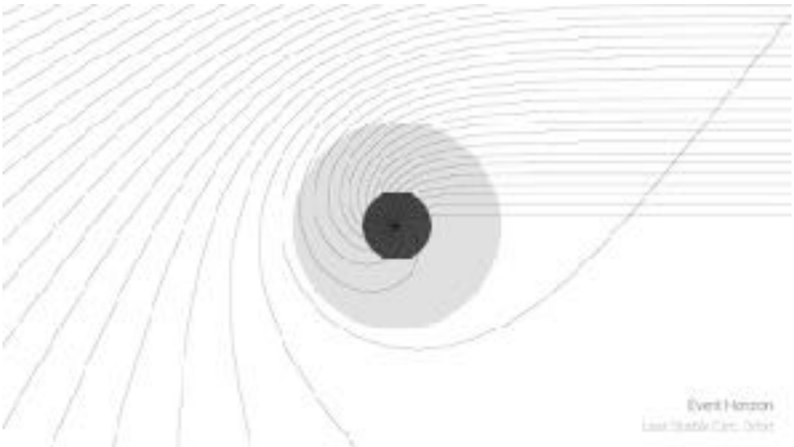
Is General Relativity ruling there?



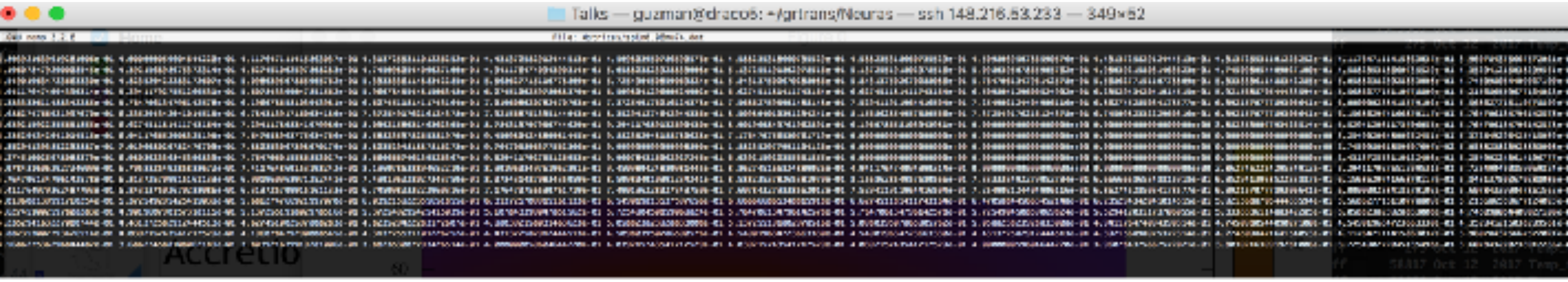


Ray tracing process

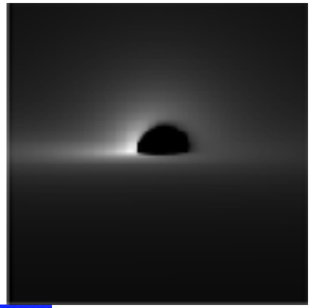


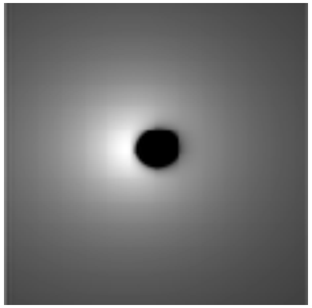


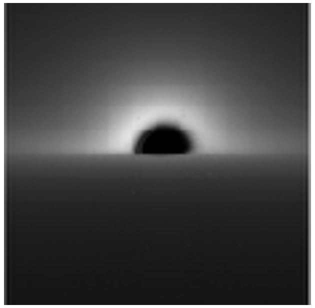
Accretion

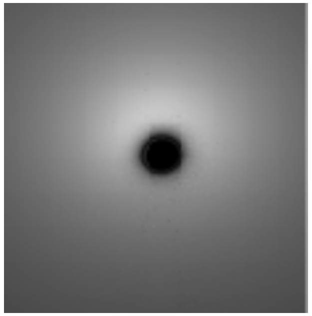


```
nu_min = 7.495e10  # <- wavelength of 4mm
nu_max = 3.527e11  # <- wavelength of 0.85 mm
```

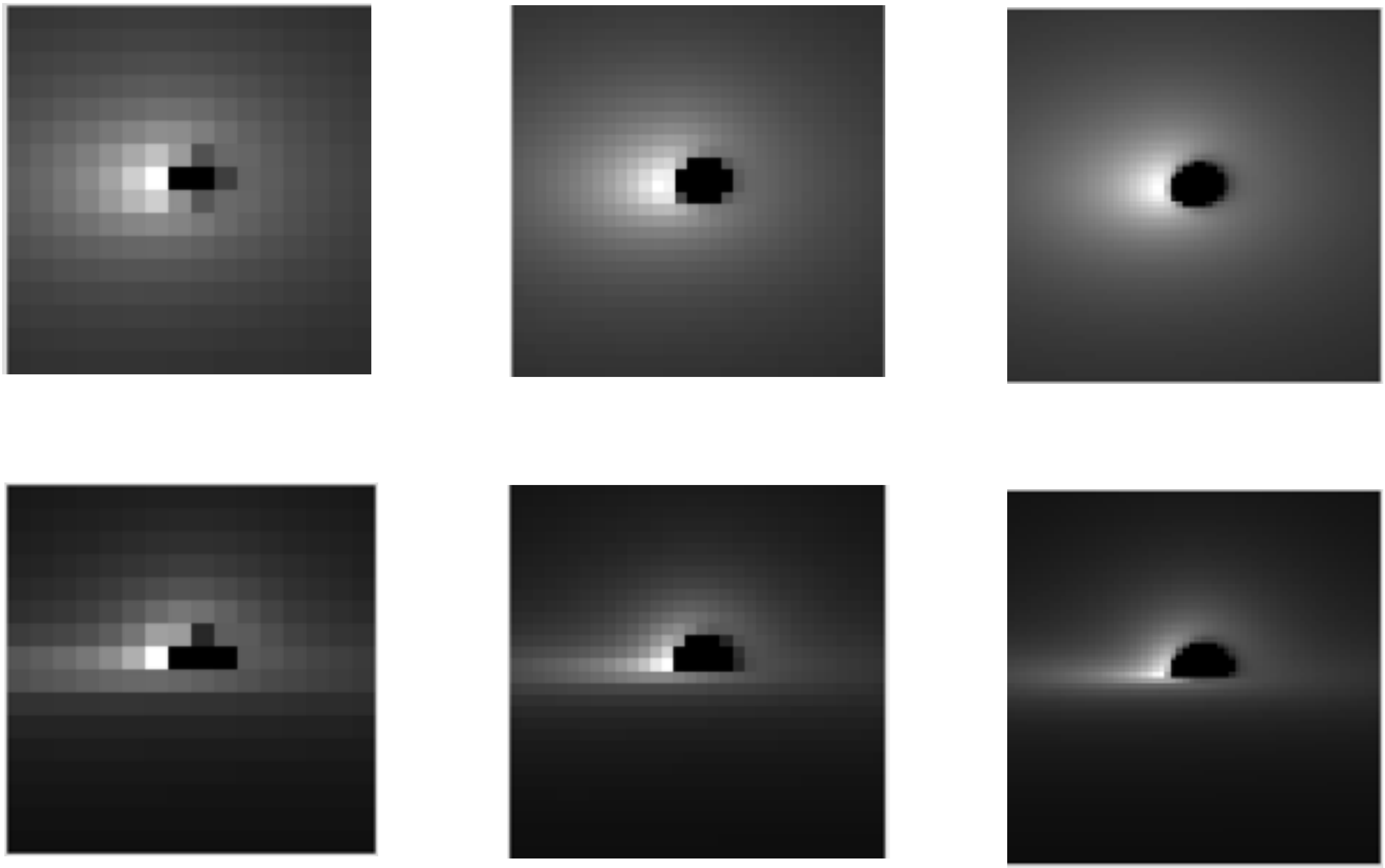




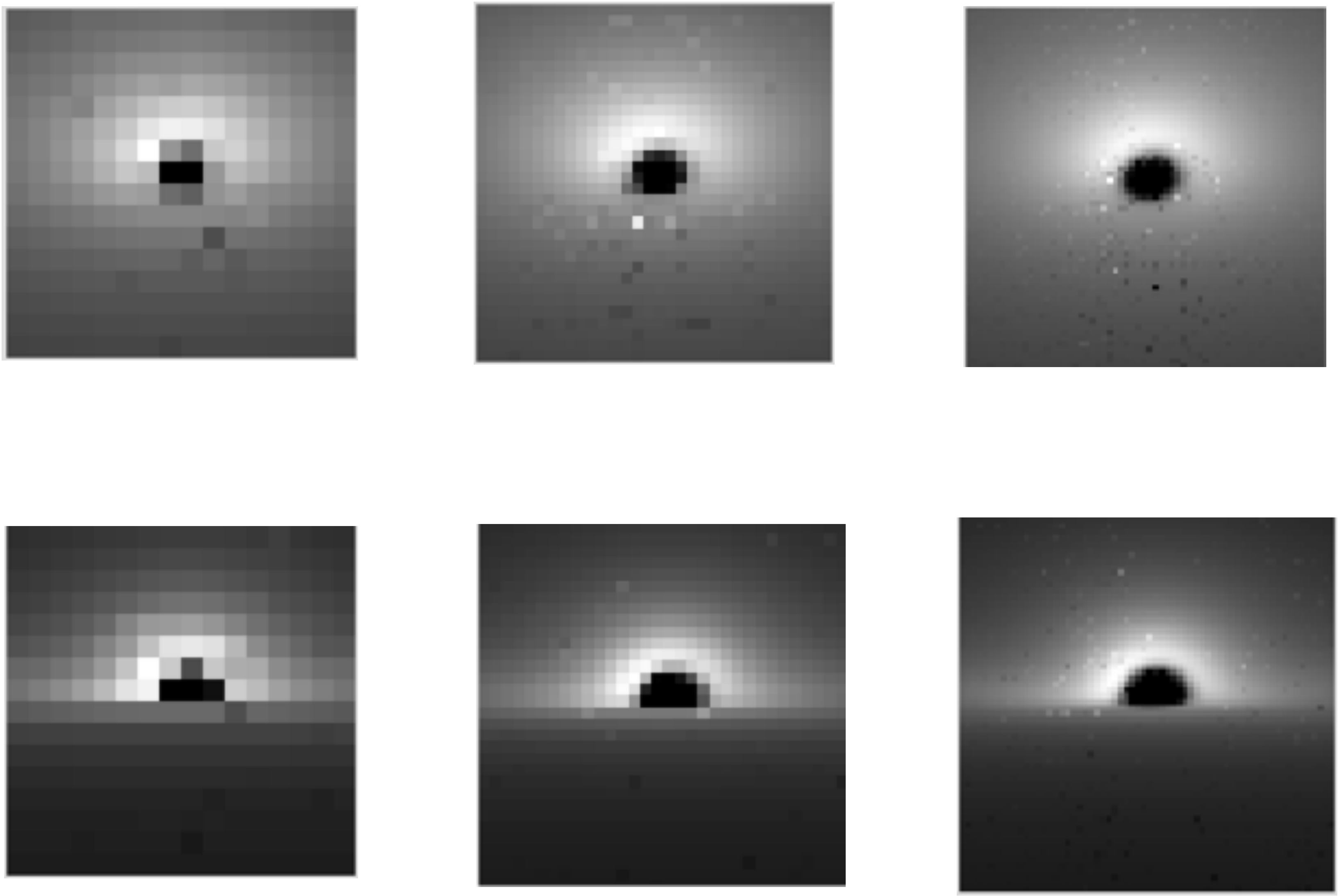




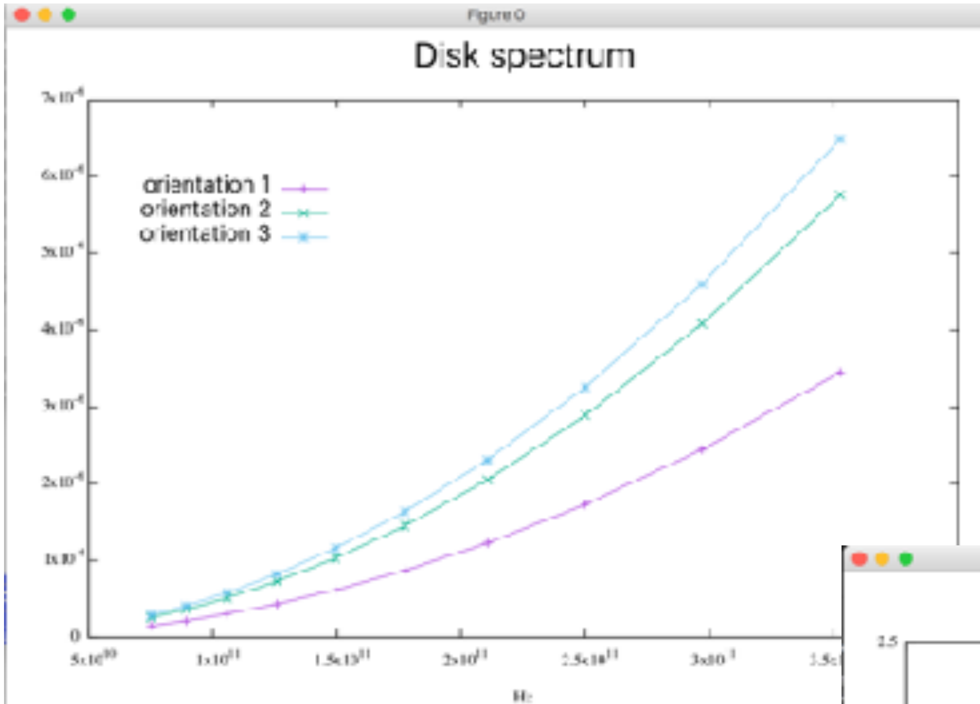
Accretion disks (s=0.85, orientation 45, 2)

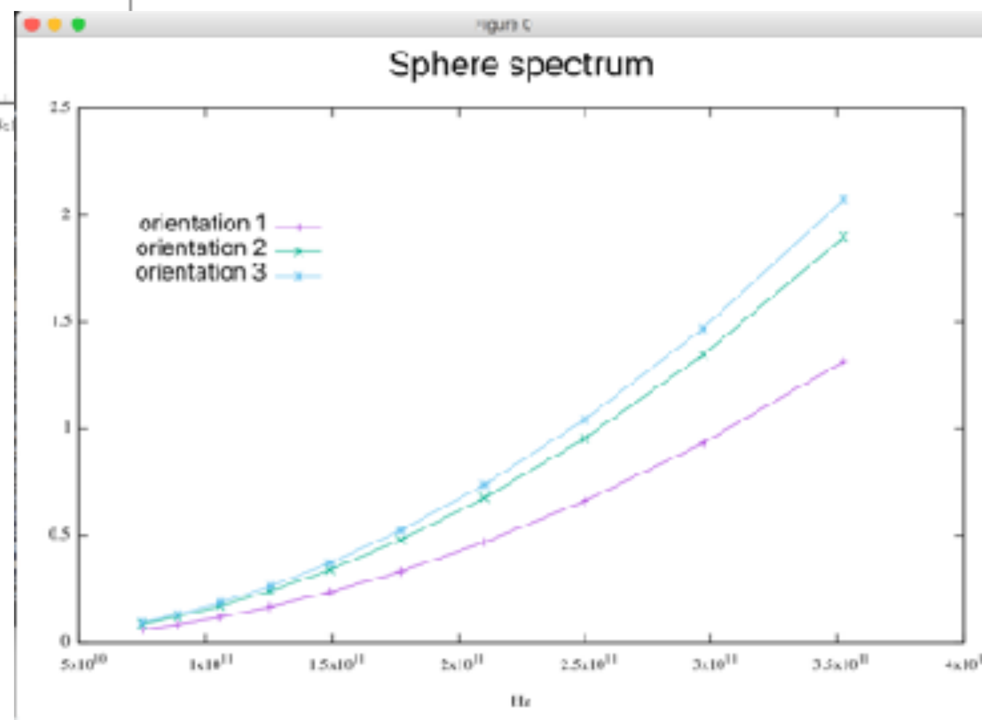


Spherical distributions



Power spectrum, s=0.95





Final comments

In order to solve an IP, one NEEDS to solve DIRECT PROBLEMS

Binary black holes have good data banks generated with PPN approximations

NS-BH Need the solution of the GRMHD equations (very expensive)

The addition of electromagnetic and neutrino radiation to GW sources
is the **multi-messenger astronomy, very expensive too… state of the art**

Wandering black holes: simulations and observations needed

Shadows of black holes does not need that much power: observational technology is needed

WHAT TO DO WITH SOME OF THIS TECHNOLOGY?

Template